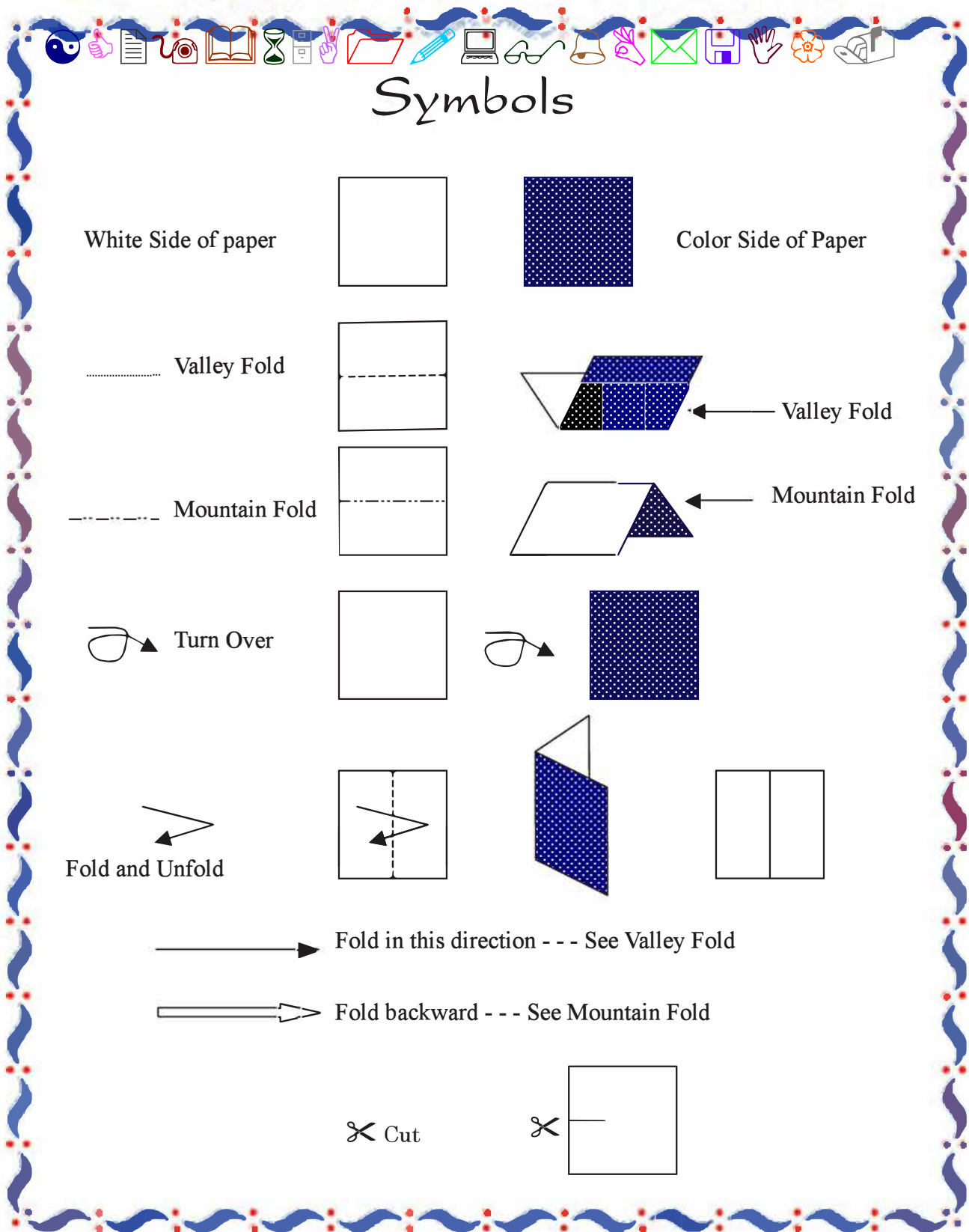
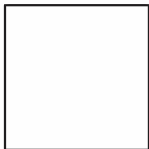
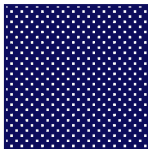
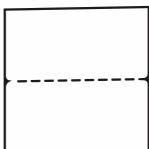
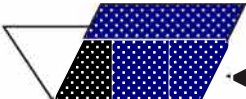
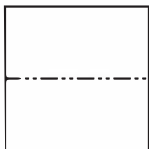


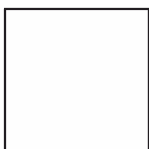
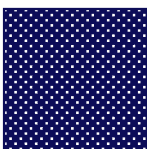
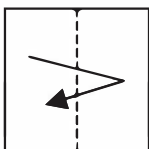

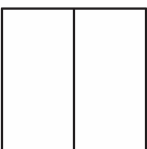




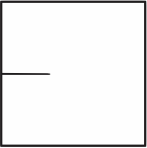


Directions for Reading Origami Diagrams

Start Here: These symbols will help you follow the instructions for reading an origami diagram. First, read the instructions on the diagram, then look ahead at the next picture to see what it will look like.



Symbols

| | | | |
|---|---|--|---|
| White Side of paper |  |  | Color Side of Paper |
| Valley Fold |  |  | ← Valley Fold |
| - - - Mountain Fold |  |  | ← Mountain Fold |
|  Turn Over |  |  | |
| Fold and Unfold |  |  |  |
| |  | Fold in this direction - - - See Valley Fold | |
| |  | Fold backward - - - See Mountain Fold | |
| |  Cut |  |  |



Paper Resources

Traditional origami paper is precut into squares ranging in different sizes, colors, and patterns. It is usually **color** on one side and white on the other side. The 6-8 inch paper squares are easier to manipulate and good for beginners. Origami paper is available in most craft stores and online.

Origami isn't Just for Squares

Teacher and budget-friendly, there are lots of *free* resources for origami. See "How to Make a Square from a Rectangle." In addition, *rectangles* can be transformed into a variety of origami models. See **Heart** and **Box** Lesson Plans.



Tips for Recycling Paper: *Reduce, Recycle Reuse, Respect*

- Start a recycling resource box in your classroom.
- Let colleagues, friends, and families know to save paper materials and help contribute to your classroom projects.
- Visit travel agencies for brochures and magazines, garage sales for calendars, thrift stores for gift wrapping paper and maps, and libraries and bookstores for promotional posters and fliers. Trim to size with a paper cutter or fold as is.

Patience, Precision, Practice: *Practice makes Progress*

Once students complete a model by practicing with recycled paper, they can personalize it by selecting a design or color that may enhance their origami project. **Nurture children's creativity.** Prior to folding, challenge students' imagination by asking them to draw their designs on a piece of paper. They can create **one-of-a-kind** artwork with computer graphics, clip art, and coloring pages from the Internet.



Math in Motion aligns with the **Common Core State Standards for Mathematics (CCSSM)**. *Progressions for the CCSSM*, K-6 Geometry, p. 13, 2013 recommends using origami as a teaching tool for geometry and other math related skills. "More advanced paper-folding (origami) tasks afford the same Mathematical practices of seeing and using structure, conjecturing and justifying conjectures. Paper folding can also illustrate many geometric concepts."

The following **Ten Teaching Techniques** will provide you with practical steps to make teaching with origami in your classroom as easy as ichi, ni, san (1, 2, 3)...all it takes is a piece (peace) of paper! Try the sample "Heart Lesson Plan." Write to share or send photos using origami in the classroom to: mathinmotion2@gmail.com. It may be posted on the [Math in Motion website](#).



Photo credit: Patricia Peña Santana

Students discover the excitement, joy, and creative connection to mathematics as they unfold a *Math in Motion* origami box.

Ten Teaching Techniques

The journey of a thousand miles begins with a single step. ~Chinese Proverb

1. **Enter the Fold.** Begin with a simple model. Place yourself where all the students can see your hands and the sample. If not everyone can see you at once, repeat the step for each side of the room. Encourage students to *observe* your demonstration of each step before they attempt it. Option: Prior to the lesson, teach one or two students in the class to help other students as needed.
2. **Unfolding the secret:** Choose larger paper to demonstrate. Your teaching model should be large enough to be seen from the back row, but not too large to manipulate. Some teaching materials can be used by recycling larger flyers, posters, or gift wrap paper cut to size 10 – 12 inch squares. Precrease your teaching model prior to demonstrating so you can pay attention to your class. **Highlight the lines on your model using different color markers** to indicate the folds so everyone can see the next step.
3. **We crease to exist:** Fold on a firm surface like a table or a book. Emphasize folding neatly and accurately. The more precise students fold, the nicer it will look. The secret to origami is in the crease. Crease each step sharply at least *three* times. The sharper you fold, the easier it will be to see and follow the guidelines on the paper for the next step.
4. **Learning to fold:** Try to ensure that your students are quiet and attentive. Students must be able to listen and follow directions in a *supportive* learning environment.
5. **Folding to learn:** Encourage students to explore the qualitative and quantitative characteristics of the materials and shapes they use. Use inquiry-based learning, a learning process that engages students by making real-world connections through exploration and high-level questioning. Questions and discussions deepen your mathematical understanding. Ask: “What can we say about the shape we see? How does this material feel?” This open-ended question approach encourages students to analyze the figure without the pressure of obtaining one right answer. It also enables the teacher to assess what the class already knows and what they may need to learn. For more ideas, see *Teacher Scripts* for the **Whale** and **Box** Lesson Plans.

6. **Square, rectangle, triangle: Make sense of mathematics.** While teaching each step to the class, introduce math concepts and vocabulary so students can experience them firsthand and learn them in context. Have students identify and label each part of the model on their paper. Younger children can trace the same areas with their fingers as they recite the parts of the figure. Write the words on the board so that they can say it and associate the term with the shape, (see *Math Concepts and Vocabulary* lists with the Lesson Plans).

7. **One fold at a time.** When describing a fold, mention the place where the fold begins and ends, or other “landmarks.” Orient your sample the same way your students are folding. Treat each step as one unit: First identify the present position and orientation of the model, perform the step, and then confirm the new position. Make sure each of your students has performed the step correctly before moving on to the next step. If you sense any uncertainty, repeat your instructions. Try to find a clearer explanation. If a step is challenging, ask students to hold their papers up to check the whole class at the same time. Make adjustments as needed. *Encourage* students to help each other.

8. **Practice makes progress.** Avoid folding the student’s model. Mistakes help our brains grow. It’s how we learn. Establish that a raised hand signals a sign for help without disturbing others. Help individual students or assign another student to assist them. If you have to perform the step on their model, unfold it and let them try it again. *Self-satisfaction* is very important. If they are still unable to perform the step, you may need to fold their model to enable them to complete it. Then, unfold it and ask them to refold it on the existing lines. With practice and patience, they will quickly develop the confidence they need to succeed.

9. **In the fold.** Be supportive and non-threatening in your instructions and corrections. Everyone learns at a different pace. Some students may seem more cautious than others and may be afraid to fail and make mistakes. Give the class as much reassurance and positive encouragement as possible.

10. **May the fold be with you. Have fun!** If you enjoy teaching and learning with origami, your students will too! It is a work in progress. Remember to be patient with yourself and take your time. Make notes of what works well and what you may need to improve each time you teach with origami.



How to Make a Square from a Rectangle



There are two easy ways to cut a square
from a rectangular sheet of paper.

Method #1

Fold the left bottom corner to the opposite side to form a triangle.

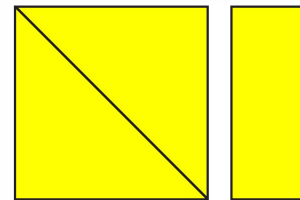
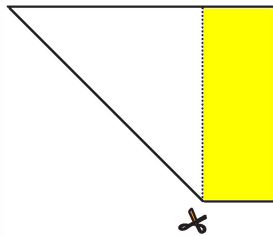
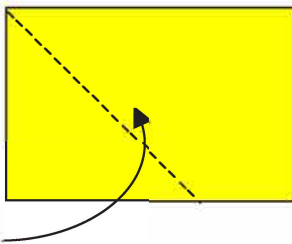
Cut along the vertical line of the rectangle. Unfold. The paper is now a square.

Alternative: *Without using scissors*, fold the rectangular portion back and forth several times. Place one hand against the triangle. Carefully separate the rectangular portion with your other hand.

The rectangular part left over can be used for making smaller squares or other models:

Make a 1) heart ♥ model or 2) a handle for the box.

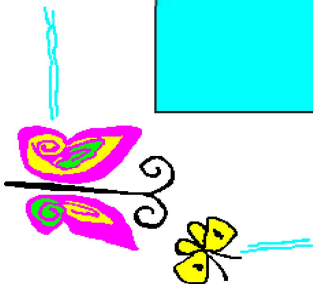
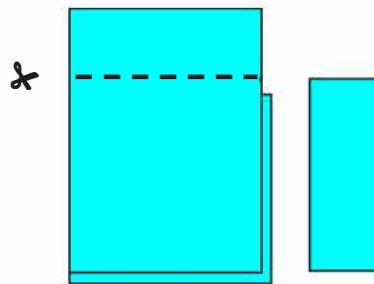
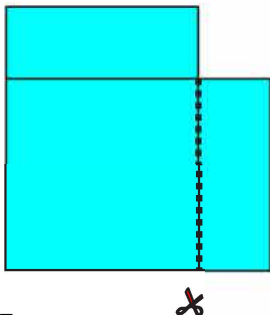
To shape a handle: Fold in half lengthwise (Book Fold). Unfold. Fold the sides to the center (Cupboard Door Fold). Staple it to the sides of the box.



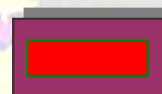
Method #2

For a square without a diagonal crease, place two identical sheets of rectangular paper as shown. Cut the bottom sheet along the raw edge of the top sheet.

The sheet underneath is now square. For a second square, turn both layers over and cut the larger sheet using the raw edge of the square as a guide.



Models Made from Rectangles



This model is easy to fold. Practice random acts of kindness. Send ☒ hearts ♥ to a sister school, nursing home, or Peace Pal 🧑.

Strand: Geometry • Spatial Sense • Language Arts • Social Studies

Materials: A red or pink rectangle, 2 x 6 inch (5.08 x 15.24 cm).

Alternative: See *How to Make a Square from a Rectangle*. Use the part left over to fold the heart model after you make a square from the 8.5 x 11 inch or A4 rectangle.

Concepts and Vocabulary:

rectangle

apex

width

congruent

right/left

pentagon

base

vertical

edge

top

isosceles right triangle

length



line of symmetry

corner

bottom

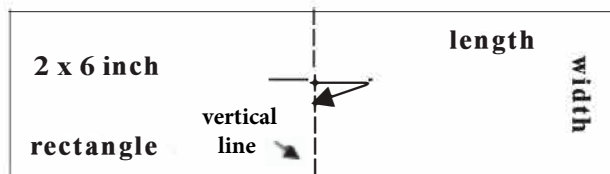
quadrilateral

Additional Activities:

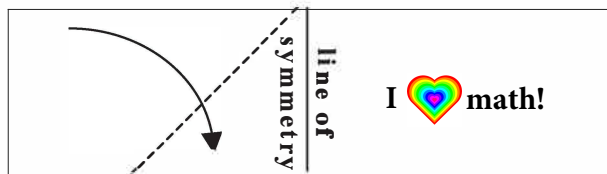
- 1 Make several Math in Motion hearts of different sizes. Teach the concept of small, smaller, smallest or big, bigger, biggest.
- 2 Make a heart for Valentine's Day. Write a heartfelt message inside. Write a poem of love and friendship and attach it to the heart.
- 3 Make up an anagram. Anagrams are letters that are rearranged to form other words or phrases like heart = earth, listen = silent, the eyes = they see, eleven plus two = twelve plus one. Can you find others? To learn more, visit: wordsmith.org/anagram/.
- 4 For St. Patrick's Day make three green hearts to form a shamrock. 
- 5 Write **love** notes. Reinforce communication between parents and children. Make it a ritual. Tape the heart to the child's shirt with a message inside. "Ask me how my day went" or "Tell me how much I mean to you." 

Material Tips: -- See "How to Make a Square from a Rectangle" to create a rectangle for the Heart model and a square for the Whale.

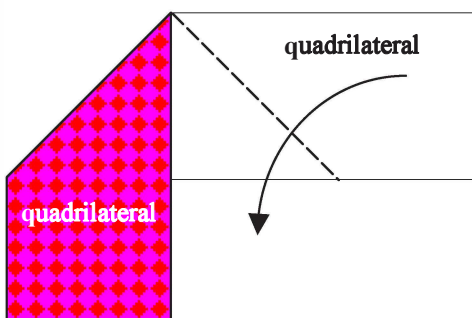
Heart



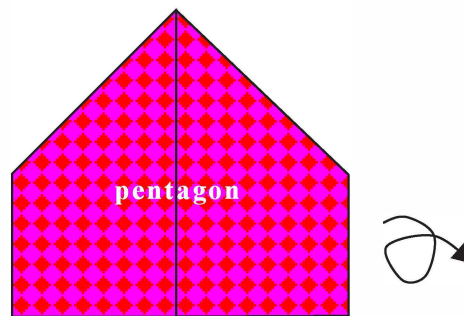
1 Start with a rectangle, white side up. Place on the table with the length (long edges) at the top and bottom. Fold in half widthwise (Book Fold). Unfold.



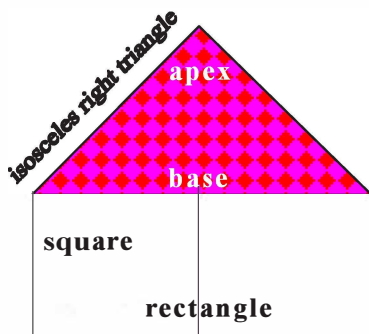
2 Fold the top left edge down along the line of symmetry (center crease).



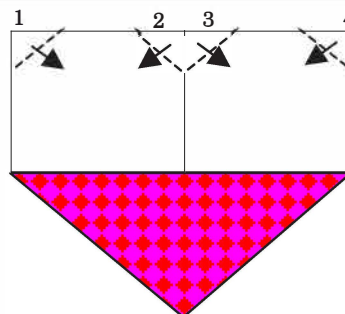
3 Fold the top right edge down to meet the center crease.



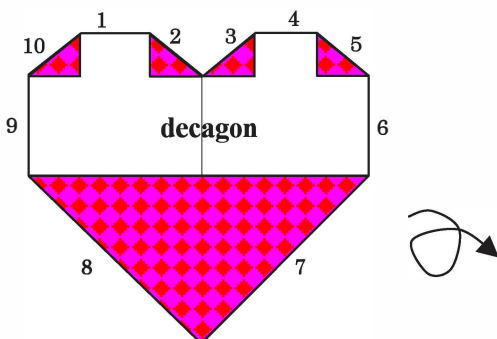
4 It looks like a tent. Turn over.



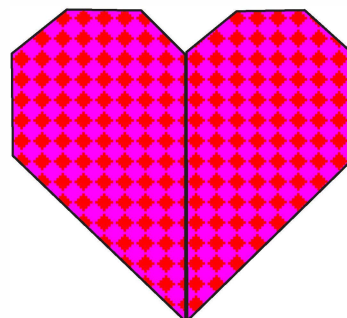
5 Now it looks like a house.



6 Turn the top around to the bottom. See the four corners at the top. Fold each corner down to form a small triangle.



7 Count the ten (10) sides -- you have a decagon! Turn over.



8 Open up your heart. Write a special message inside. Make up a heartfelt saying. I love MATH! Teaching is a work of heart. I ♥ you!

Models Made from Squares



This fishy model is easy and good for beginners. Have a whale of a time!

FUN FACTS: The whistle of the blue whale is the loudest sound produced by any animal at 188 decibels.

Strand: Geometry • Spatial Sense • Patterns • Connections • Science

Materials: A 6-inch or larger square (15 cm), start white side up.

Concepts & Vocabulary:

square
congruent
right

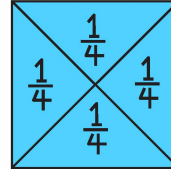
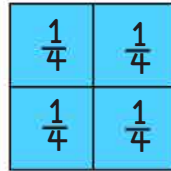
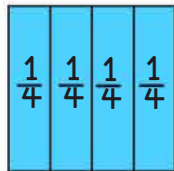
quadrilateral
vertical
left

triangle
center
point

scalene triangle
isosceles right triangle
line of symmetry

Additional Activities:

1. How many ways can a square be divided into 4 equal parts?



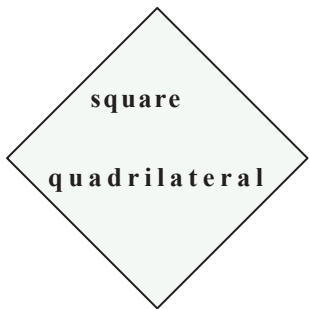
2. Make a whale as a Father's Day card for a "Whale of a Dad." Use as name tags for the first day of school "a school of fish" or "Back to School" Night.
3. Did you know whales sing? Nobody knows for sure, but perhaps they are singing the blues. Listen to Paul Winter's, "Lullaby from the Great Mother Whale for the Baby Seal Pups," *Concert for the Earth, Live at the United Nations*. Younger children will enjoy singing "Baby Beluga" with *Raffi in Concert*.
4. Read *A Thousand Pails of Water* by R. Roy (K-2), *Whale of a Tale* by Barbara Pearl (PreK-2), *Humphrey*, *The Wayward Whale* by Ernest Callenbach (K-6) and other stories about fish-like mammals (dolphins, manatees, porpoises, seals, and walruses). Be a **Changemaker**. Raise awareness about our oceans, plastic pollution and its effect on sea animals. See the lesson plan from the Wildlife Movement at: [Save the Oceans](#) by Bethany Stahl. To adopt a whale, contact:



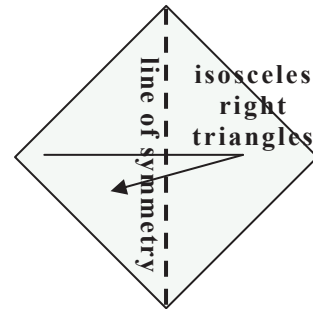
Greenpeace
1436 U Street, N.W.
Washington, D.C. 20009
1-800-456-4029
www.greenpeace.org

Save the Whales
1192 Waring Street
Seaside, CA 93955
1-800-942-5365
www.savethewhales.org

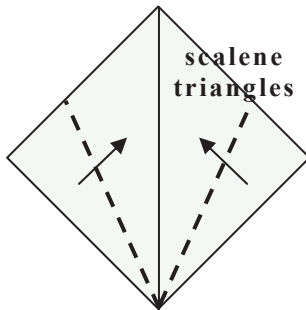
Whale



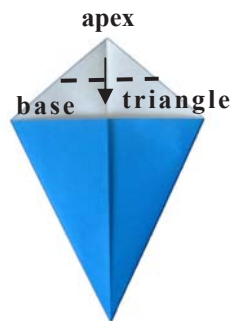
- 1 Place a 6-inch square sheet of paper on the table so it looks like a diamond.



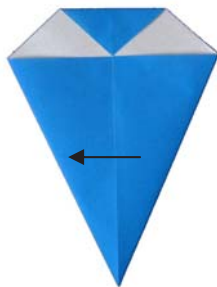
- 2 Fold the left point over to meet the right point (fold in half). Unfold.



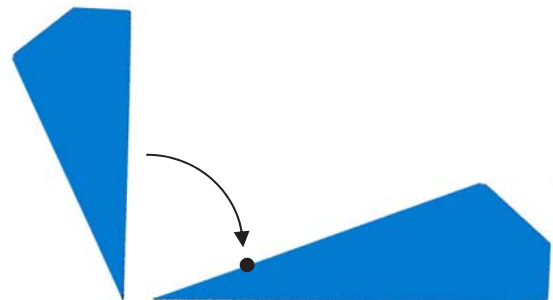
- 3 Fold the lower left and right sides to meet at the vertical line of symmetry.



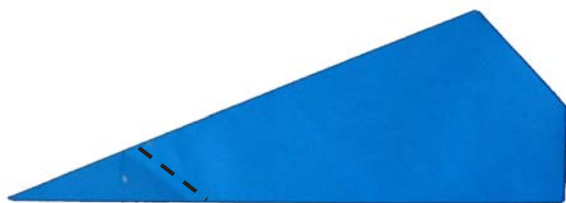
- 4 It looks like a kite. Now fold the apex (top point) down to the base line to form a small triangle.



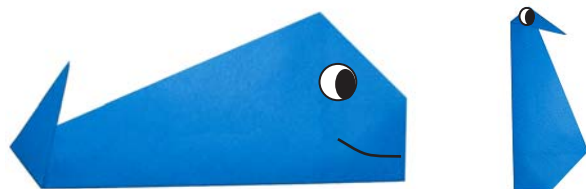
- 5 Fold the right side over to meet the left side (fold in half).



- 6 Put your finger on the bottom point as you turn the whale sideways.

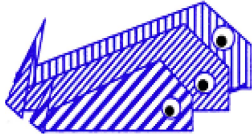


- 7 Fold the end point up to make a tail.



- 8 Draw a happy face. Have a whale of a day!
For a surprise, open the whale, rotate to stand and create a penguin instead.

How to Fold a Whale



Use **Inquiry-based** learning for a student-centered teaching method that encourages students to ask questions and investigate real-world problems. In this type of learning environment, students are actively engaged in the learning process and are given the opportunity to explore.

1. Place a square sheet of paper on the table so it looks like a diamond shape.

What shape is the paper?

Square.

How many sides does a square have?

4 sides.

What else can you say about the sides?

The sides all have the same length.

Another name for a four sided figure is **quadrilateral**.

Can you think of other quadrilaterals?

Rectangle, parallelogram, rhombus, trapezoid.

Name other quadrilaterals in the room.

Book, chalkboard, desk, table, window.

2. Fold the right point over to the left point. Unfold.

What is the vertical line?

Line of Symmetry.

What is a line of symmetry?

A line of folding so that the two halves of a figure match.

What shape did it create?

Triangles.

What kind of triangles?

Isosceles right triangles.

What is an isosceles right triangle?

It has two sides of equal length and a square corner.

3. Fold the lower left and right sides to meet the center crease. Unfold the paper and discover more triangles.

What kind of triangles?

Scalene triangles.

What are scalene triangles?

They have no **congruent** sides.

How many triangles are there?

Six. Refold the model.

What does it look like?

It looks like a kite or an ice cream cone.

4. Fold the apex (top point) down to the base (widest part of the kite) to form a small triangle ▲. Unfold.

How many triangles are there?

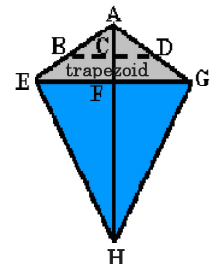
11 triangles. See: ABD, ABC, ADC, AEG, AEF, AGF, AEH, AGH, EFH, GFH, EHG.

What other shapes do you see on the kite?

Trapezoid- BDGE.
Quadrilateral -AGHE.

What can you say about triangles EFH and GFH?

They are **congruent** triangles.
Can you find others?

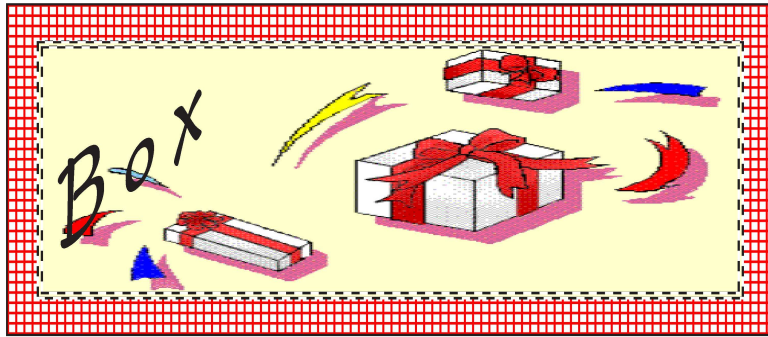


5. Refold the model. Fold the **right** side over to meet the **left** side.

6. Put your finger on the bottom point as you turn the whale sideways.

7. Fold the **point** up to make a tail. Draw a happy face. Have a whale of a day! 😊

Congruent: In geometry, two figures or objects are congruent if they have the same shape and size, or if one has the same shape and size as the mirror image of the other.



This model was traditionally known as the “magazine box” because it was made out of magazine covers. Make two boxes the same size and connect them together.

FUN FACTS: Paul Weinberg, a grandfather had an exhibit of his origami boxes made out of magazine covers at a New York library. He called the display, “Literary Origami.”

Strand: Fractions • Geometry • Measurement • Art

Materials: 8.5 x 11 inch rectangle (color copy paper)



Alternative: For a decorative box, recycle the front and back covers of a magazine.

Concepts and Vocabulary:

Fractions

| | |
|------------|------|
| whole | 1 |
| halves | 1/2 |
| quarters | 1/4 |
| eighths | 1/8 |
| sixteenths | 1/16 |

Geometry

| | |
|----------------|--------------------------|
| rectangle | perpendicular lines |
| quadrilateral | isosceles right triangle |
| triangle | octagon |
| length | width |
| parallel lines | line of symmetry |

***Additional Activities:**

1. Calculate the area and volume of the Math in Motion box.

AREA

A. What is the area of the base of the box?
 Measure the length and width. $A = lw$
 Area is measured in square units.

1) Round your answer to the nearest 10th.
 $A = 23.4 \text{ in.}^2$ ($5.5 \times 4.25 = 23.375$)

2) Round to the nearest whole number.
 $A = 23 \text{ square inches}$

VOLUME

B. What is the volume of the box?
 Measure the length, width and height. $V = lwh$
 Volume is measured in cubic units.

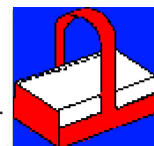
1) Round your answer to the nearest 10th.
 $V = 49.7 \text{ in.}^3$ ($5.5 \times 4.25 \times 2.125 = 49.671$)

2) Round to the nearest whole number.
 $V = 50 \text{ cubic inches}$

2. **Challenge Question:** 1) Find the **surface area** of the box. **Surface Area** = $2lw + 2lh + 2wh$ or $SA = 2(lw + hl + hw)$. Since this box has no top, use the formula: $SA = lw + 2lh + 2wh$

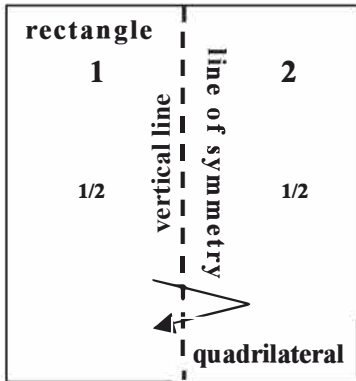
3. Calculate the answers for the **area, volume and surface area** in metric units.

4. Make a basket. Fold a piece of paper lengthwise and staple a handle to the sides.

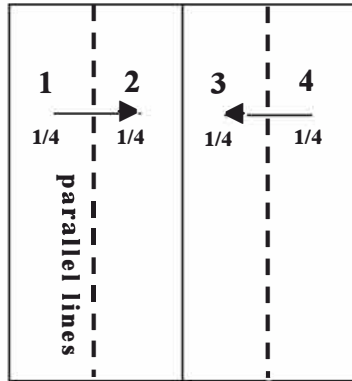


5. Write a haiku poem. Place it inside the box or basket to give as a gift.

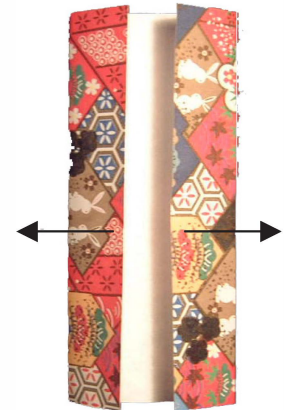
Box



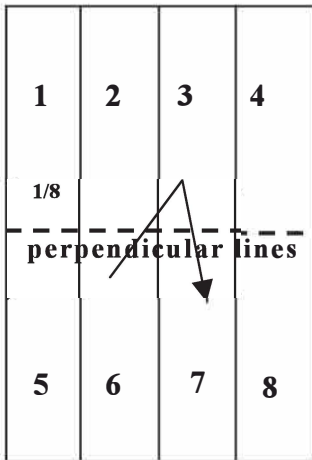
1 Start with a rectangle, white side up. Fold in half lengthwise (Book Fold). Unfold.



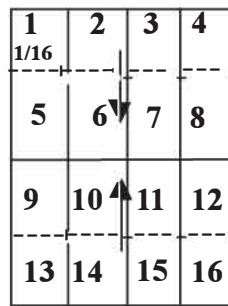
2 Bring longer edges to meet at the center crease (Cupboard Door Fold).



3 Unfold.



4 Fold in half widthwise (Book Fold). Unfold.



5 Bring shorter edges (width) to meet at the center crease. This time leave the folds in.

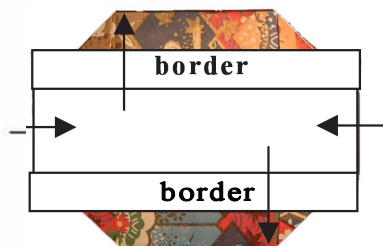
isosceles triangle or 45° right triangle



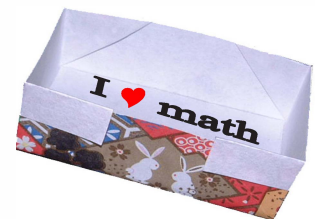
6 At each corner, bring the folded edge to lie along the nearest vertical crease to form a triangle. **Note:** Folded edges do NOT reach the centerline.



7 Fold the rectangular borders, one up and one down, as far as you can over the triangles to lock them down in place.



8 Place a thumb under each of the rectangular borders and pull them apart. The ends of the model will fold upward to form the sides of a box. Pinch the corners of the box to shape.



9 Voila! A finished box! Make another box the same size for a lid. Connect the boxes together.



How to Fold a Box

*I have all the **right** angles.*

Explore the patterns on the paper each time students fold the steps and label it. Younger children can trace the same place with their fingers or crayons as they identify it. Give students time to explore and discover math concepts and vocabulary. The questions on the left side are in italics. Some suggested responses are on the right side.

1. Fold the whole rectangle in half lengthwise. Unfold.

What is a rectangle?

A **quadrilateral** that has four right angles.

What is a quadrilateral?

A quadrilateral is a geometric figure with four sides.

What is a right angle?

A right angle measures 90° .

Name other quadrilaterals.

Square, parallelogram, rhombus, trapezoid.

What kind of line is it?

A **vertical line and a line of symmetry**.

What is a line of symmetry?

A line of folding so that the two parts of a figure match (congruent).

How many rectangles do you see all together?

Three.

Write the **numbers** 1 and 2 in each section of the paper model (see Step #1). Younger children can draw stars or apples to correspond with the numbers. Explore concepts like **one-to-one correspondence** and to distinguish between **right and left**. Older children can write **fractions** in each part. Write the fraction $\frac{1}{2}$.

Discuss parts of a fraction. In $\frac{1}{2}$, 1 is the numerator and 2 is the denominator.

What is a numerator?

2

Represents how many parts of the whole you have.

What is a denominator?

Represents how many equal parts the whole is divided into.

2. Bring longer edges (length) to meet at the center crease (Cupboard Door Fold). Unfold.

How many equal parts are there?

Four. Write in each part $\frac{1}{4}$.

What kind of line do you see?

Vertical lines. 4

What is another name for these lines?

Parallel lines.

What are parallel lines?

Lines that never intersect (cross or meet).

3. Fold in half widthwise. Unfold.

How many equal parts is the paper divided into?

Eight. Write in each part $\frac{1}{8}$.

What kind of lines do you see?

Perpendicular lines. Trace the lines.

What are perpendicular lines?

Lines that intersect and form right angles, 90° .

4. Bring shorter edges to meet at the center crease. Unfold. Discover a grid. The number grid offers many opportunities to explore and investigate number patterns and their relationships: place value, counting, symbol recognition, comparing greater than and less than ($<$ $>$), multiples of 2, 3, 4 and powers of 2 and 4. Analyze the data and graph the results.

Unfold the paper. How many parts do you see?

Sixteen. Write in each part $\frac{1}{16}$.

5. Place the paper with the width at the top (see diagram step #5). Write the numbers 1-16 in each part. Younger children can practice counting forward and backward, skip counting by 2s and 4s. Practice adding or multiplying the rows and columns. Calculate mentally or with paper and pencil. Check your work with a calculator.

Put a square around the odd numbers.

1 3, 5, 7, 9, 11, 13, 15.

Circle the even numbers.

2 4, 6, 8, 10, 12, 14, 16.

Write the even and odd numbers up to 50 or 100.

Discover even + even numbers will always give an even number. **Example:** $2 + 4 = 6$.

Even + odd numbers will always give an odd number. **Example:** $2 + 3 = 5$.

Odd + odd numbers will always give an even number. **Example:** $3 + 5 = 8$.

Allow students to experiment, make predictions, test, justify and prove their work.

Ask students to draw or write about how they would explain their answers. Share observations.

Discuss prime and composite numbers.

A prime number is a whole number that is greater than 1 and has *exactly* two factors, 1 and itself. One is neither prime nor composite.

What is a prime number?

Place the letter "P" over all the prime numbers.

2, 3, 5, 7, 11, 13.

What is unique about the number 2?

Two is the only even prime number.

What is a composite number?

A composite number is a whole number that is greater than one and has *more* than two factors.

Place the letter "C" over all the composite numbers. Extend to 100.

4, 6, 8, 9, 10, 12, 14, 15, 16.

Investigate number patterns. See *vertical* and *diagonal* number patterns.

What is the pattern for 1, 5, 9, 13?

They all increase by the number 4.

Name or write the next 10 numbers

17, 21, 25, 29, 33, 37, 41, 45, 49, and 53.

in the series. Look for other patterns.

Compare numbers. Explore greater than and less than. Write five number sentences. Use the greater than $>$ and less than $<$ symbols. Example: Seven is less than nine. Write $7 < 9$.

Nine is greater than seven. Write $9 > 7$.

6. Refold step #5. At each corner, bring the folded edge to lie along the nearest crease line.

Note: The folded edges DO NOT reach the crosswise centerline.

What shape do you see?

Triangle. Isosceles right triangle.

How many sides does the whole shape have?

Eight.

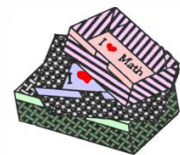
What is the name of the shape?

Octagon.

Is the octagon a regular or irregular shape?

The octagon is irregular.

If all angles are equal and all sides are equal, then it is **regular**, otherwise it is **irregular**.



7. Fold the rectangular borders over the triangles. Repeat on the other side.

8. GENTLY open the sides to form a box. Pinch the corners to shape into a box.

9. Now you have a finished box. Make another box the same size for a top and connect them together.

