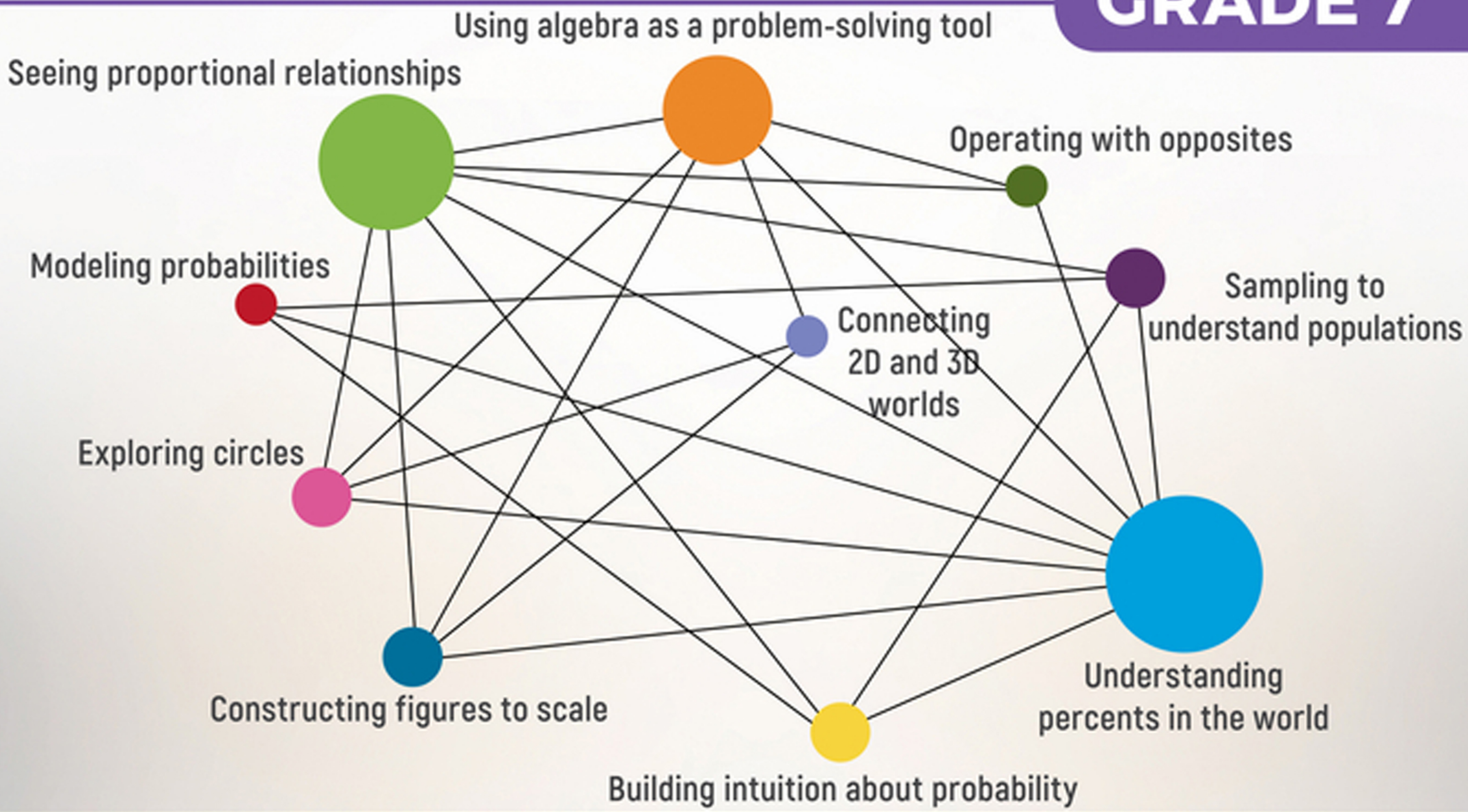
Visualizing and Investigating **Big Ideas**

mindset mathematics

GRADE 7



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What's the Chance of That?

Snapshot

Students find and compare the theoretical and experimental probabilities for tossing a cube onto different colored regions of a painting, connecting probability to area.

Connection to CCSS
7.SP.7, 7.SP.6, 7.SP.5

Agenda

Activity	Time	Description/Prompt	Materials
Launch	10 min	Show students the What's the Chance of That? image and ask, If I were to toss a bead onto this image, which color would it be most (and least) likely to land on? Why? Make connections between area and probability.	What's the Chance of That? sheet, to display
Explore	30+ min	Groups explore the probability of a cube landing on each color of the What's the Chance of That? painting by analyzing the painting and conducting experiments. Students develop ways of finding and expressing probability. Groups compare their experimental and theoretical probabilities.	• What's the Chance of That? sheet (full color or wireframe), one per group • Centimeter cube or other small object for tossing, one per group • Books or other objects for forming a border around the image, for each group • Optional: colors

Activity	Time	Description/Prompt	Materials
Discuss	15+ min	Discuss the probability of a cube landing on each color and the methods students developed for finding these probabilities. Distinguish between theoretical and experimental methods and compare the results of the two types of strategies. Make connections between equivalent representations of probability, and connect probability to area.	What's the Chance of That? sheet, to display
Extend	30+ min	Groups design their own paintings for playing this tossing game. Challenge students to design a painting not based on 100 squares. Groups then find the probability of a cube landing on each color.	1″ grid paper (see appendix), scissors, colors, and tape, for each group

To the Teacher

This activity is designed to support students in connecting percents, fractions, probability, and area. In Big Idea 5, we introduced probability with spinners to build intuition and later moved toward quantifying probability with bags of countable cubes. Here, we integrate the ideas that area can be a measure of probability and that area can be countable if regions are decomposed into equal-size units. The painting we have adapted for students to explore is based on an underlying—and invisible—10 × 10 grid composed of 100 equal-size square units. The structure of this painting is similar to the one students explored when building ideas about percents in Big Idea 4, so students should have some experience with decomposing to quantify the portion each color represents. One reason connecting area to probability is particularly powerful is that area can serve as a model for probability. Even when area does not determine probability, as it does with a spinner, area can be used as a way of modeling probability, particularly for compound events. This is not the focus of this activity, but we are planting the seed of this idea here.

In preparing for this activity, you'll want to try experimenting with tossing a small object onto the painting to determine what students will need to make this work in your space. Centimeter cubes, beads, or other tossing objects need to be small enough to fit in a square unit, though they may often land on the borders between regions. Support students in thinking about what they will do if their object doesn't land completely inside one color. Any consistent approach is fine. We recommend providing students with some books to construct walls around the painting to keep the tossed object from bouncing out. Test out how high these borders need to be with the object you are tossing.

Activity

Launch

Launch the activity by showing students the What's the Chance of That? image on the document camera. Tell students to imagine that this painting is a game board. Ask, If you tossed a bead onto this board, which color would be most likely for the bead to land on? Which color would be least likely? Why? Give students a chance to turn and talk to a partner. Discuss students' reasoning, and draw attention to thinking that focuses on the area of each color.

Pose the question, What are the chances of a centimeter cube landing on each color?

Explore

Provide each group with the What's the Chance of That? sheet, a centimeter cube or other small object, and objects (such as books) to form a border around the painting. You can provide students with either the full-color or wireframe version of the sheet; if you use the wireframe version, you'll want to continue to display the color version on the document camera as a reference and provide access to colors.

Students explore the following questions:

- What is the probability of the cube landing on each color?
- How do you know?
- What are the different ways you can find the probability?
- What ways can you come up with to express the probabilities you find?
- What is the relationship between the experimental and theoretical probabilities you found?

Encourage students to both experiment with tossing the cube and analyze the painting itself to find the probabilities of landing on each color. For testing out their ideas, show students how to use the books to make a box around the painting to keep the cube in when tossed. Students will need to develop a way for dealing with instances when the cube lands on a boundary between two colors.

Discuss

Show the What's the Chance of That? sheet on the document camera as a reference. You can use this to mark up the ways that students analyzed the painting. Discuss the following questions:

- What is the probability of a cube landing on each color? How do you know?
- What strategies did you develop for finding the probability? Which strategies were experimental? Which were theoretical?
- What different ways can you come up with to express the probabilities you find?
- What is the relationship between the experimental and theoretical probabilities you found?

Highlight the ways students express probability, including fractions, decimals, percents, and language, and make connections between equivalent forms. Draw attention to the ways that students can use area as a model for probability, and connect this to the thinking students did with spinners in the Visualize activity from Big Idea 5.

Extend

Invite groups to design their own painting for this kind of game and find the probabilities of a bead landing on each of the colors they used. Provide students with 1″ grid paper (see appendix), scissors, tape, colors, and a bead or other object for tossing. Challenge students to design a painting that does not have 100 squares as the underlying framework. They will need to use their understandings of fractions, decimals, and percents to help them reason about area and probability.

Look-Fors

- **Are students analyzing the painting?** Much of the focus of Big Idea 5 was on building intuition about probability through experimentation. However, we want students to see that analyzing the possible outcomes is a more reliable and accurate way of predicting the probability of an event. As you observe students trying to find the probability of the cube landing on each color, look for groups that might be only experimenting. Ask students questions to support their thinking about analysis, such as, How do you know your probabilities are accurate? Is there any way to be more precise? How do you know from looking at the painting that one outcome is more likely than another? Is there a way to say exactly how much more likely? How could you use the painting as a tool for finding the probability of landing on each color?

- **Are students connecting area to probability?** One of the features of the painting we have designed is that it rests on an underlying 10 × 10 grid, with 100 equal-size squares. The probability of landing on a color depends on how large the regions of each color are, and the 100 squares in this painting make that area countable. To do so, students will need to decompose the painting into squares they can see. As you observe students working, look for whether students are decomposing the colored regions into equal-size pieces to quantify their area and connect that to the probability of the cube landing in that space. Students will also need to compose the separate regions that are the same color to find the total probability of landing on a given color, not just the probability of landing on one section of the painting. You might ask, Which color is it more likely for the cube to land on? How do you know? If students can name that it is the size of the region, or the area, that matters, then you can ask, If the area tells you how likely it is, then how can the area help you find the probability?
- **Do students see the difference between theoretical and experimental probabilities?** We began discussing the difference between theoretical and experimental probabilities in Big Idea 5, and in this activity, we want students to explore both and compare them. When the class discusses their findings, be sure to draw attention to the differences in the experimental probabilities that different groups generated for the same color and ask why this variation exists. You might ask, Why did you all get different results? Does this mean we made mistakes? Similarly, you might ask questions about the consistency of the theoretical probabilities that students found (or, if groups found different answers, you might ask the same questions that you did about experimental probability), such as, Why in this case did you all get the same results? Why is this different from experimenting? You might also ask students why we might use each kind of probability. In this activity, experimenting can serve as a check for finding theoretical probability, but in some cases, it may be very difficult to find theoretical probability, and what happens in real life is simply easier to see, as when skill and chance are combined.

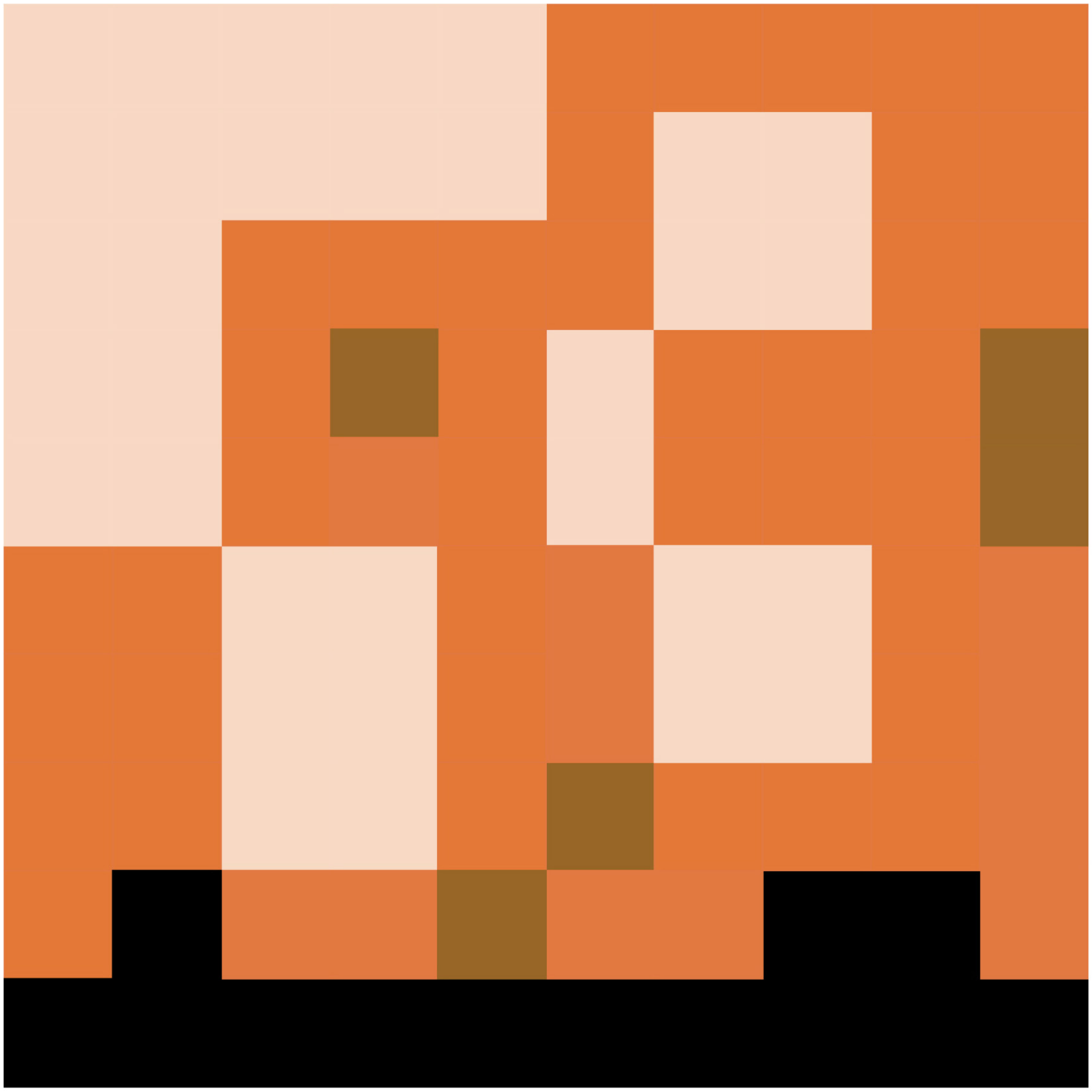
- **How are students expressing probability?** We pick up this thread from the Investigate activity from Big Idea 5 and have designed this activity to provide a different kind of context for quantifying theoretical probability. Students may use fractions, words, ratios, decimals, or percents to express probability, and the underlying structure of the painting allows students to navigate any of these forms without focusing on calculation. Ask students, Why are you choosing to use this form? How does it help you compare probabilities? All forms can be accurate, but fractions, words, or ratios that do not reference the same size of whole can be difficult to compare. If the fractions all use the same denominator, however, they can be just as easy to compare as decimals and percents. When discussing the probabilities, be sure to make connections between equivalent forms of the same probability and ask, Which forms do you think are easiest to compare? Why?

Reflect

How is area related to probability?

What's the Chance of That?

Based on Leon Polk Smith's *Accent Black* (1949)



What’s the Chance of That? (Wireframe)

Based on Leon Polk Smith’s *Accent Black* (1949)