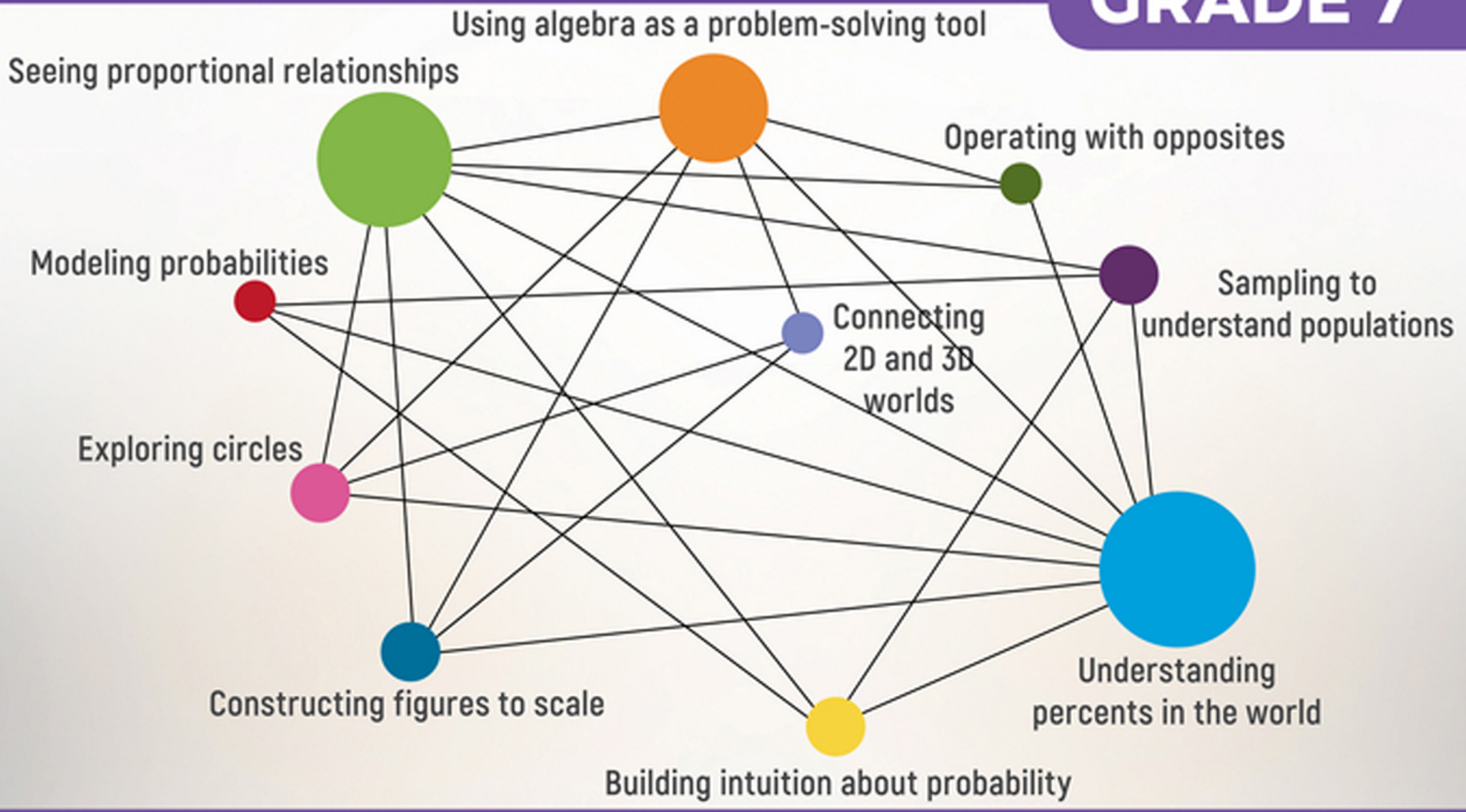
Visualizing and Investigating Big Ideas

mindset mathematics

GRADE 7



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What's in the Bag?

Snapshot

Students explore bags of colored cubes to predict their contents. Groups try to name how likely it is to draw a blue cube from the bag, inventing ways of quantifying probability.

Connection to CCSS
7.SP.5, 7.SP.6, 7.SP.7

Agenda

Activity	Time	Description/Prompt	Materials
Launch	10 min	Show students one of the bags of cubes and tell them that inside are colored cubes, but we don't know how many or what colors. Invite students to draw out one cube and use that to make a prediction about the contents of the bag. Repeat this process to show how more data increases confidence in predictions.	Prepared bag of cubes, any one from the set made for the class
Explore	30+ min	Groups explore multiple bags of cubes and try to make a conjecture about the contents of each. Students may draw out one cube at a time, replacing it before drawing the next.	Several prepared bags of colored cubes (see To the Teacher section for details)
Discuss	20–30 min	For each bag, discuss groups' conjectures about the contents, how students arrived at their ideas, and how these predictions compare. Reveal the contents of each bag and compare the contents to the conjectures. Discuss the methods that proved most accurate.	Several prepared bags of colored cubes (see To the Teacher section for details)

Activity	Time	Description/Prompt	Materials
Explore	45+ min	Students play a game with the bags in which a player wins when they draw a blue cube. Groups explore the question, What are the chances of winning for each bag? How can we put the bags in order from least to most likely to win?	• Several prepared bags of colored cubes (see To the Teacher section for details) • Bag Cards, to match the prepared bags, for each group
Discuss	20+ min	Draw a line from impossible to certain and invite groups to add Bag Cards one at a time to the line, justifying the placement of each. Support students in developing quantitative ways of justifying how likely it is to draw a blue cube from each bag. Compare the results to the experiments students conducted earlier and discuss why they are not identical.	• Bag Cards, one class set • Tape, magnets, or pushpins for posting the cards

To the Teacher

To prepare for this activity, you'll need to assemble several bags of cubes. The bags need to be opaque and easy to reach into without looking at the contents, such as drawstring bags, paper lunch bags, or socks. You'll want to have at least one bag per group, though you could have more for the class to explore. Construct the bags so that they range in the total number of cubes and the number of blue cubes, which, in the second half of the investigation, will be the winning cubes. You'll want to choose some combinations for which the probabilities of drawing a blue cube are similar, and you'll want one bag where the probability is 1 (certain, or 100%) and one bag where it is 0 (impossible, or 0%). Here are some suggested bags for you to select from:

15 cubes: 15 blue
12 cubes: 11 blue and 1 red
20 cubes: 15 blue, 4 yellow, and 1 red
10 cubes: 5 blue and 5 yellow
12 cubes: 5 blue, 4 red, and 3 yellow
20 cubes: 8 blue, 8 red, and 4 yellow

15 cubes: 3 blue, 6 red, and 6 yellow
8 cubes: 2 blue, 4 yellow, and 2 red
10 cubes: 1 blue, 6 yellow, and 3 red
14 cubes: 8 red and 6 yellow

These distributions are provided on cards for you to cut out so that students can use them in the second half of the investigation as they try to sort them by the probability of drawing a blue block. Notice that each bag is given a letter label on the cards. You'll want each of your bags to have a letter label so that students can refer to the bags and compare conjectures for the same bag. If you make your own bags, you'll need to make your own cards to match.

The big push in this activity is figuring out ways to quantify probability. In the previous activities, students built intuition about and language for probability, with a focus on more, less, and equally likely outcomes. This activity bridges students into more formal and precise ways of describing probability by posing the question, *How* likely is it?

Activity

Launch

Launch the activity by showing students one of the bags of cubes and telling them that inside are a bunch of colored cubes. Tell students that, without looking, we don't know what exactly is inside the bag. We don't know how many cubes there are or what color they are. In today's investigation, the goal is to figure out what's in the bag, and to do that we are allowed to take one cube out at a time, look at it, and then put it back.

Invite one student to draw a cube out of the bag and show it to the class before replacing it. Ask the class, What do you think is in the bag? Why? Give students a chance to turn and talk to a partner, then take some ideas from students. Students will likely say that seeing one block isn't enough to know what is in the bag. If so, invite another student to draw another cube to show to the class and replace it. Repeat the question, What do you think is in the bag? Why? Discuss how this second draw gives more information, but not enough to be confident.

Tell students that today they are going to be exploring a bunch of bags and using this one way of gathering data—drawing and replacing cubes—to develop conjectures about what's in each bag.

Explore

Provide one bag of cubes (see the To the Teacher section) to each group. Tell students that each bag contains cubes. Ask groups to develop their best conjecture about the number of each color cube that is in the bag. To do this, groups may draw out cubes one at a time, but after each time, they must replace the cube in the bag. Note that students will likely feel the cubes to count how many there are; this is entirely appropriate as long as they are not looking at the contents.

Rotate the bags from group to group so that each group has a chance to develop ideas about each bag. Groups should keep a record of the data they collect for each bag, and label that data so they know which bag it came from.

Discuss

Discuss the following questions for each bag in turn:

- What are your conjectures about the contents of the bag?
- How did you arrive at your conjecture?
- How confident are you in your conjecture? What could you do to increase your confidence?
- How do our conjectures compare? Which do you think are most accurate? Why?

As you discuss the different bags, students may suggest combining their data to make their conjectures more accurate. If they do, support students in trying this and then comparing the results to the groups' individual conjectures.

After you discuss each bag, open the bag and show the class the contents. The document camera works well for this. Discuss the following:

- Which conjectures were closest?
- What does this tell us about effective methods?

After you have discussed all of the bags, ask students these questions:

- What methods helped us get closest when making conjectures about the contents of the bags?
- Why do you think these worked?

Explore

Tell students that they are going to explore a game with these bags. In this game, players take turns drawing a cube out of the bag and replacing it. A player wins if they draw out a blue cube.

Groups explore the following questions:

- What is the chance of winning for each bag? How do you know?
- How can we put these bags in order from most likely to win to least likely to win?

Provide each group with access to the bags and a set of Bag Cards to match the bags your class explored. Groups can use these cards for information and to literally arrange them in order from least to most likely.

Discuss

Draw a horizontal line on a board, labeled *impossible* on one end and *certain* on the other, where students can post the cards from least to most likely. Invite each group to place one card on the board and to explain why they are placing it there. As each group places a card, they need to explain its position on the line and its position relative to the other cards. You might ask these questions:

- How did you decide where to place the card?
- Why do you think it is more (or less) likely than the card next to it?
- How likely is it that a person will draw a blue cube from that bag? How do you know?

The central goal of this discussion is to support students in finding ways to describe probability with numbers, whether they use language such as *40%*, $\frac{4}{10}$, or *4 out of 10*. When discussing bags with a similar probability of drawing a blue cube, students will need to work to justify one or the other as more likely. These cases encourage quantifying probability in some way. Spend time discussing these cases to draw out ways of accurately comparing the probabilities of drawing a blue cube.

Ask students, How do these probabilities compare to the data you collected earlier? Depending on the sample size, these probabilities should be close to, but not exactly, what students found. You might take this opportunity to name these as *experimental probability,* or the probability of an event found through simulating

that event, and *theoretical probability,* or the probability of an event found through calculating the possible outcomes. These two forms of probability should be close, which is to say that the theoretical probability predicts what is likely to happen in life, but they are rarely identical. You might ask students why they think that is.

Look-Fors

- **How are students connecting their data to their conjectures?** As students collect data about each bag, they have the opportunity to make connections between that data and a prediction about the bag's contents. Students may try to do this quite literally, by drawing out 10 cubes one at a time and then predicting there are 10 cubes composed of their exact results. This strategy can enable students to form one kind of prediction, provided there are actually 10 cubes in the bag, but a more precise prediction could be made by drawing cubes 20 or 30 times. If they do this, students will need to think proportionally about the contents. If they feel that there are 15 cubes in the bag and draw cubes 30 times, they might halve their results to make a prediction. This would be more accurate, and could also lead to an impossible prediction that there are $4\frac{1}{2}$ blue cubes. If you notice this kind of thinking, be sure to draw attention to the value in the larger number of trials and the proportional reasoning the students were employing. You might then ask, Do you really think there are $4\frac{1}{2}$ blue cubes in this bag? If that's not possible, what prediction could you make from your data that is possible?
- **Are students finding chance by experimenting or by analyzing the bag's contents?** In the second half of the investigation, we invite students to compare the probabilities of drawing a blue cube out of each bag. Our intention is to encourage students to develop ways to quantify probability, and students may do this in one of two ways. Students may simply conduct the experiments again, collecting data to see which bag wins more often. Or students may analyze the content of each bag and compare the chances of drawing a blue cube based on the number of cubes in the bag. Both can lead to defensible conclusions, but only an analytic approach can lead to consistent, precise results. Experiments change depending on the particular trials and how many you choose to conduct. To encourage students toward an analytic approach, you might ask students who are experimenting, Do you think your results will always be true? How confident are you that one bag is more likely than another? You might encourage students to focus on two bags for which you

know the probabilities of drawing a blue cube are close to one another. The only way to be confident about these bags is to focus on the contents.

- **How are students describing the chances of drawing a blue cube?** There are many ways students might quantify the chances of drawing a blue cube. Students may use fractions, decimals, ratios, percents, or language, such as "1 out of 10" or "impossible." All of these are valid ways of describing probability. The key is to make connections between these various forms so that students can see that they are equivalent, and they can make decisions about which form they think makes the most sense for expressing or comparing probability. For instance, "2 out of 8" or $\frac{5}{12}$ can be completely accurate ways of expressing probability, and they have the added benefit of potentially naming the number of cubes in the bag as well as how many are blue. But these forms can make *comparing* probability challenging; decimals or percents make this task much easier, even if they obscure the number of cubes in each bag. Encourage students to quantify the probability by asking, How likely is it? Then ask students, How will you compare these two bags to know with confidence which one is more likely to win?

Reflect

Design a bag of colored cubes. What is the probability of drawing each color in your bag? How do you know?

Bag Cards

Bag A	Bag B
20 cubes	**8 cubes**
8 blue	2 blue
8 red	2 red
4 yellow	4 yellow
Bag C	**Bag D**
15 cubes	**10 cubes**
15 blue	5 blue
0 red	0 red
0 yellow	5 yellow
Bag E	**Bag F**
15 cubes	**12 cubes**
3 blue	11 blue
6 red	1 red
6 yellow	0 yellow

Bag G	Bag H
10 cubes	**14 cubes**
1 blue	0 blue
3 red	8 red
6 yellow	6 yellow
Bag I	**Bag J**
12 cubes	**20 cubes**
5 blue	15 blue
4 red	1 red
3 yellow	4 yellow