

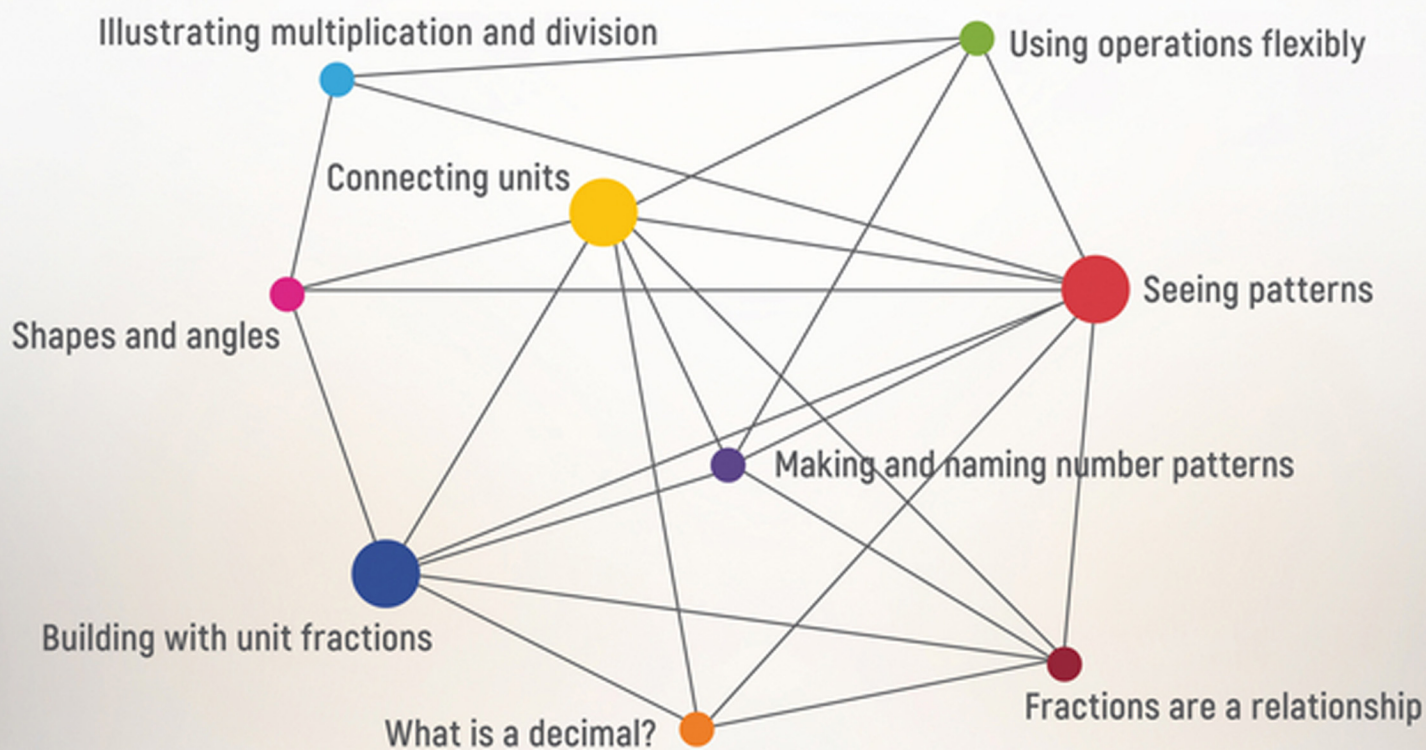
Visualizing
and Investigating
Big Ideas



mindset

mathematics

GRADE 4



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Supply Parade

Snapshot

Students extend their work with multistep problem solving and estimation to figure out how many boxes of pencils the class would use in a school year.



Connection to CCSS
4.OA.3

Agenda

Activity	Time	Description/Prompt	Materials
Launch	5 min	Orient students toward all the consumable supplies in their classroom. Pose the question, How many boxes of 12 pencils do you think we use in one school year?	A box of 12 pencils to show students (optional)
Explore	30+ min	In pairs or small groups, students work to develop a strategy for estimating the number of boxes of pencils the class will use in one year. Students gather the information they think they need to arrive at an estimate, construct a plan, and record their work to share with other.	Posters and markers for each group
Discuss	20 min	Groups present their estimates, and the class asks questions to determine how convincing the methods presented are. Students discuss the decisions they made in developing their methods. The class comes to some agreement about what the best estimate likely is.	Student posters
Extend	20+ min	Students figure out how much money the school could save by buying the class pencils from a store offering a cheaper price, rather than from a more expensive store.	Office or classroom supply catalogs (optional)

To the Teacher

This investigation focuses on crafting a strategy for estimating how many boxes of pencils the class will use in a school year. We chose this particular supply because pencils are used in every classroom, often supplied by schools, and frequently lost, and they come in packages. However, you could substitute any number of items if you think they would be of particular interest or relevance in your class. You could investigate any consumable common in your classroom and provided by the school. This could include paper, markers or other colors, soap or hand sanitizer, paper towels, tissues, staples, or milk. You will want to avoid investigating items students are responsible for purchasing, because students' financial capacity to buy supplies should not be a factor under exploration. You may want to take this opportunity to have students investigate their use of a supply of which you'd like to promote conservation, including paper, paper towels, or food.

Different items pose different challenges. Some items are used in great quantity, like paper, which can lead to working with very large numbers. Other items, like milk or hand sanitizer, cannot be counted individually. These have to be measured in some way, and this measurement will pose an additional layer of challenge. With hand sanitizer, for example, one would need to figure out how many pumps one can get from the bottle; with milk, one would need to decide what units make sense to use and how to measure consumption. This may mean that students need to physically investigate (say, by pumping lots of hand sanitizer), which requires resources. You'll want to consider these challenges when choosing an item to investigate.

The final consideration is packaging. Pencils typically come in boxes of 12, which is a relatively small number (unlike paper, which comes in reams of 500 pages). Asking about boxes of pencils opens the door for students to choose to use division or repeated subtraction, or to think multiplicatively when moving from individual pencils to packages. If you choose to substitute a different item for pencils, consider whether it makes mathematical sense for students to think about individual items or packages given the numbers that might be involved.

Activity

Launch

Launch this investigation by pointing out to students some of the supplies they use in the classroom, particularly the kinds of things that get consumed and have to be purchased again and again. Every classroom uses lots of things. You might ask students to generate examples, too. To buy all these supplies, the school has to know how much classrooms need. One thing we use a lot of is pencils. Pencils get used, lost, and broken all the time, and we always need new ones. How many pencils do you think we use in one year? You might ask students to turn and talk to a partner and come up with a quick ballpark estimate. The pencils we buy come in boxes of 12. (Show students a box of 12 pencils, if you have one.) Tell students that today you and your group will come up with a strategy for estimating how many boxes of pencils we use in this classroom in one school year. At the end of this investigation, groups will be asked to share a poster with their estimate and how they came up with it. You'll need to convince us that your estimate makes sense.

Explore

Students should work in pairs or small groups to develop a strategy to estimate how many boxes of pencils your classroom uses in one school year. Remember that each box holds 12 pencils. Start the exploration with an opportunity for groups to make a plan together before diving in. What information would you need? How could you get it? Students might want to collect some data by investigating any pencil cups or trays you have in your class, counting pencils in desks, or surveying students about how many pencils they have in their backpacks. Encourage them to collect and organize any information they think would be helpful. You might have groups share ideas for getting started or information they think they need, before sending groups off to work on the task.

Be sure to provide groups with a poster and markers. Encourage students to use color coding to help make the parts of their work clear. Posters should show their entire process and be convincing that their estimate makes sense.

Discuss

Gather students together to share their different estimates and, more important, the strategies they developed for generating these estimates. Students listening to the

methods shared should be ready to ask questions to clarify or challenge the processes each group has used. Pose these questions:

- Does their strategy make sense? Can you follow all the steps?
- Could you explain what they did to someone else?
- Is their method convincing?
- Do you disagree with any part of the process? If so, why?
- Is there something they could have done to make their estimate more accurate? If so, what and why?

As students share, draw attention to the decisions students have made about what operations, information, and tools to use and the reasoning that underlies these decisions. The initial decisions students made likely had a big impact on the pathways they crafted for solving this problem. For instance, groups that thought about boxes of pencils first probably never had to divide, whereas those that thought about individual pencils had to think about how to form boxes with those pencils later.

At the close of the discussion, ask students to look again at all the posters. Thinking about all of these different methods and estimates, what do you think is the most accurate estimate? It could be one of the estimates offered or something in between. Have students share their reasoning. This is a good time to help students make sense of their estimates by making them visual. Students may try to put the numbers in order and choose the middle, or maybe combine the different class estimates and try to find a middle number. This type of thinking may not be in your fourth-grade standards; however, it is coming up in grade six. Allowing students flexibility in their thinking, and helping them organize their estimates visually are productive mathematical practices.

Extend

Pencils are not very expensive—not nearly as expensive, say, as tables or computers. But the cost of pencils can add up over time. Different stores offer slightly different prices for pencils. If one store sells boxes of pencils for \$3.00 and another store sells the same boxes for \$2.50, how much money would your school save by buying the pencils for your class at the cheaper store this year?

The numbers used in this extension matter. If you have not yet worked with decimals, you'll want to stick with prices that are easy to work with intuitively, like

the ones given or whole dollars. However, you can adjust these prices to incorporate work with decimals. We suggest that in this case, you choose numbers that can easily add repeatedly and whose difference is straightforward to calculate, like \$2.25 and \$2.75.

As a further extension, you might ask, What else could be purchased for our classroom with these savings? You could give students access to an office supply catalog or other teacher resource catalog so that they can think creatively about what the savings could buy.

Look-Fors

- **What assumptions are students making as they begin? What data are they collecting?** In order to make a justifiable estimate, students will need to first come up with an idea of how many pencils are used in a shorter time frame, perhaps a week. Alternatively, they might simply figure out how many pencils are being used now and make some assumption about how quickly they need to be replaced. Each of these is a critical decision that will have a big impact on the estimates generated. It is worth probing the reasoning behind these initial estimates and assumptions to make sure students feel that these make sense and are convincing. If these aren't convincing, the estimates built on them won't be either.
- **How are students recording their process and keeping track of their intermediate calculations?** Students need an organizational system for solving a problem with so many potential parts. As you talk to students in the midst of their work, encourage them to think about how to track their work and how the poster could be a useful tool.
- **Are students thinking about the school year or the calendar year?** Student may need access to a school calendar to help them think about the difference and find out how long the school year actually is in your district.
- **Are students attending to and challenging one another's work in the discussion?** Students should be actively making sense of the different ways that other classmates arrived at their estimates and asking questions. Students should point out parts of the strategy that are not convincing and suggest things the groups could do to strengthen their estimates.

Reflect

How did you decide what operations to use to solve this problem?

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Cockcroft, W. H. (1982). *Mathematics counts: Report of inquiry into the teaching of mathematics in schools*. London: Her Majesty's Stationery Office.