Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach

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Equity is a concept that is often measured in terms of test scores, with educators looking for equal test scores among students of different cultural groups, social classes or sexes. In this article the term 'relational equity' is proposed to describe equitable relations in classrooms; relations that include students treating each other with respect and responsibility. This concept will be illustrated through the results of a four-year study of different mathematics teaching approaches, conducted in three Californian high schools. In one of the schools—a diverse, urban high school—students achieved at higher levels, learned good behaviour, and learned to respect students from different cultural groups, social classes, ability levels and sexes. In addition, differences in attainment between different cultural groups were eliminated in some cases and reduced in all others. Importantly, the goals of high achievement and equity were achieved in tandem through a mixed-ability mathematics approach that is not used or well known in the UK.

Introduction

As the world becomes increasingly more globalized and communication across cultures an everyday part of life, schools need to renew their attention to the opportunities students receive to learn about effective communication, cultural appreciation, and respect. In considering the impact of globalization for education, many scholars unite in calling for more opportunities for students to learn to think analytically and creatively, work across disciplinary boundaries, and interact productively with individuals from different cultural backgrounds (Gardner, 2004, p. 254). I contend, as have others (Cogan & Derricott, 1988; Steiner-Khamsi *et al.*, 2002), that one of the goals of schools should be to produce citizens who treat each other with respect, who value the contributions of others with whom they interact,

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irrespective of their race, class or gender, and who act with a sense of justice in considering the needs of others in society. A first step towards producing citizens who act in such ways is the creation of classrooms in which students learn to act in such ways, for we know that students learn a lot more than subject knowledge in their school classrooms. Equal educational opportunity, a concept that I will refer to as 'equity' (Post, 2004), is often measured in terms of test scores, with educators looking for equal test scores among students of different cultural groups, social classes or genders (Gutiérrez, 2002), but I would like to ask the question, what would it mean to create classrooms in which students are learning to *act* equitably?

In this article I propose the term 'relational equity' to describe equitable relations in classrooms; relations that include students treating each other with respect and considering different viewpoints fairly, and I suggest that one route to achieving such relations emanates from a change in the ways core school subjects are taught and learned (see also Matthews & Sweeney, 1997). Eisner reminds us of the simple but seemingly elusive fact that the goal of school is not to do well in school but to do well in life (Eisner, 2004). The notion of equity that I consider in this article moves the focus away from school outcomes, such as tests scores, and onto the relations, ways of acting and ways of being that students learn in school and that they take with them into their lives. I extend the notion of equity to the relations between students with the assumption that the ways students learn to treat each other and the respect they learn to form for each other will impact on the opportunities they extend to others in their lives in and beyond school.

The larger study from which this article will draw is a four-year study of different mathematics teaching approaches, conducted in the USA as a follow-up to the study of different mathematics approaches I conducted in England (1997a, 2002). In the USA I worked with a team of doctoral students to follow approximately 700 students over four years at different high schools. We used a range of qualitative and quantitative research methods to assess the impact of the different teaching approaches upon the students' learning. One of the surprising results of the study was the success of one school in particular and the excellent social relations that developed among students. In a diverse, urban high school, students achieved at high levels, learned good behavior, and learned to respect students from different cultural groups, social classes and genders. Importantly, the goals of high achievement and equity were achieved in tandem through a mixed-ability mathematics approach that is not used or well known in the UK. Our close monitoring of the students over four years, which included over 600 hours of classroom observations, enabled us to analyse and understand how the school brought about such achievements. It is commonly believed that students will learn respect for different people and cultures if they have discussions about such issues in Personal, Social and Health Education (PSHE) classes or read diverse forms of literature in English or Social Studies classes. I propose that all subjects have something to contribute in the promotion of equity and that mathematics, often regarded as the most abstract subject, removed from responsibilities of cultural or social awareness, has an important contribution to make.

Other articles have described the mathematics approach of the successful school and the ways that it resulted in high-level mathematical understanding (Boaler & Staples, forthcoming). This article will reflect upon the ways that this approach also helped students learn respect, responsibility and socially just behaviour (Matthews, 2003). At a time when levels of indiscipline in English schools are causing concern among politicians and teachers (Baker, 2004), it seems important for schools to learn about approaches that encourage high attainment, responsibility and cultural appreciation, hand in hand.

Literature Review

As educators, we need to consider where students may learn such principles and values as respect and responsibility for others in their crowded school days. Two possible sources for such learning are the curriculum, formal and informal, and the pedagogy of classrooms. The curriculum home for such principles and practices in England and Wales has tended to fall under a banner of 'citizenship education' (Gearon, n.d., p. 8; Qualifications and Curriculum Authority [QCA], 1998; National Curriculum Council [NCC], 1990, 2006). In 1990 two reports were published within months of each other, the Speaker's Commission on Citizenship's (1990) *Encouraging citizenship* followed by the National Curriculum Council's (1990) *Education for citizenship* (Derricott, 1998, p. 25). These documents laid the path for the 2004 NCC proposals on citizenship and the decision that Citizenship should become a core subject, along with English, mathematics, science, information and communications technology (ICT) and physical education. The NCC Citizenship documents for each key stage include attention to student relationships and the respect pupils form for differences between people. At Key Stage 3, for example, it is stated that pupils should learn to 'think about, express and explain views that are not their own' (NCC, 2006)—a key feature of the mathematics approach that will be presented in this article. These goals are highlighted alongside such issues as legal and human rights and responsibilities and discussions of 'political, spiritual, moral, social and cultural issues and problems' (NCC, 2006).

But where should such important learning goals fit in the packed curriculum that schools usually offer? The knowledge that is defined, regarding parliament, human rights and responsibilities, for example, fits naturally into a social studies curriculum and it is not surprising that one strategy in the promotion of citizenship education has been a coalition with history and geography, two subjects that themselves 'fight for space' in the curriculum (Derricott, 1998, p. 24). But the attitudes, abilities and values that are listed under the other areas of citizenship, such as communication skills, problem-solving skills and respect for others, could arguably be seen as the responsibility of a range of subject departments. While this point is noted by the NCC, the advice and resources they offer for the encouragement of respect and responsibility concern PSHE or fall between the different curriculum areas. The NCC does not convey the idea that core subjects such as mathematics and science

have a role to play in the encouragement of such personal qualities. Indeed, the location of such important values within a curriculum area that is un-assessed and marginalized (Derricott, 1998, p. 27) may lead to their neglect and under-representation. Derricott (1998) reports that programmes such as world studies, peace studies, development studies, multicultural education and human rights education have to 'fight for space at the margins of an already cluttered curriculum' (p. 24) and that their success rests upon the work of particular committed individuals who have the time and the status to give them a place. This means that key values and practices that students need to learn may not be considered the responsibility of teachers of mathematics, science and other subjects.

Nel Noddings has produced an important body of work, over a number of years, arguing that schools need to pay more attention to the teaching of care and compassion among students (see, for example, Noddings, 1992, 2005). In a recent collection (Noddings, 2005) she and other scholars consider ways to educate citizens 'for global awareness' and Noddings argues that all subjects have a contribution to make in educating citizens. 'Even mathematics', she argues, 'the most closely defined of all subjects—can include a study of birthrates, incomes, comparative health data, war casualties, the cost of social programs, systems of taxation, and appropriate means for collecting and evaluating such data' (2005, p. 123). In arguing that mathematics be used to consider sources of inequality Noddings adds to a group of scholars who are concerned with equity and who maintain a similar position. Frankenstein (1989, 1990), Gutstein (2002) and others have designed approaches that encourage students to consider issues of equity in the world, to understand and care about matters of social justice, such as pay structures and government spending and to equip students with mathematical methods and ideas that enable them to take on injustices (Frankenstein, 1989, 1990; Gutstein *et al.*, 1997). Noddings, Gutstein, Frankenstein and their collaborators argue that mathematics teachers have responsibilities to educate for equity and they focus their attention on the ways that curricular examples may promote cultural and social awareness. In this article I will consider other ways in which mathematics teaching and learning can promote equity and fairness. In the USA there is much support for the use of curriculum materials that are designed to connect with students' cultures, in different subject areas (Tharp & Gallimore, 1988; Gutstein *et al.*, 1997; Lee, 2001). The use of 'cultural modeling' (Lee, 2001, p. 100) has been shown to be effective but the school that is the subject of this article did not explicitly relate mathematics to the students' home cultures. Their impressive achievements came about through a mathematics curriculum that was largely abstract but that enabled multiple methods, solution paths and points of discussion and negotiation.

Bernstein and Bourdieu both extend discussions of equity beyond the curriculum materials that are presented to the pedagogical practices that are encouraged in classrooms. Through theories of pedagogic relay and recontextualisation Bernstein (2000) demonstrates the ways in which particular cultural values are embedded in pedagogy in ways that often remain invisible. By remaining unseen to participants, pedagogy reinforces particular aspects of a dominant culture, thereby marginalizing

other cultures (Morais *et al.*, 1992; Lerman *et al.*, 2003). Theories of cultural and social capital (Bourdieu, 1982, 1986) provide support for this perspective. In Bourdieu's terms, students enter all mathematics classrooms with a habitus that provides the lens for interpreting and acting in that field (Bourdieu, 1986). Zevenbergen (2000, 2005) has claimed that mathematics classrooms that are reform-oriented convey a different set of valued practices than traditional classrooms and that the habitus needs to be adjusted to allow for other ways of acting and interpreting. According to Zevenbergen (1996), students have to constitute a new habitus in reform classrooms, aligning with the new ways of acting and working, in order to be seen as successful students. Both Bourdieu and Bernstein offer ways of interpreting mathematics classrooms that shed light on the reasons why newer, reformed pedagogies may not result in more equitable achievement (Lubienski, 2000). Their attention to the aspects of newer pedagogies that are often implicit, or hidden, are also helpful in interpreting the equitable outcomes of Railside school, where teachers enacted reform pedagogies but also carefully taught the 'learning practices' (Cohen & Ball, 2001) that students would need in order to be successful. The explicit attention they paid to the ways in which students would need to work, including starting projects by outlining what successful work looked like and stopping students to highlight a productive way of working (Boaler & Staples, forthcoming), were important features of the equitable teaching they employed.

Most scholars of classroom equity in the USA concentrate their efforts upon curriculum materials and the content of different courses that may serve an enlightening purpose for students. But European researchers, such as Bourdieu, Bernstein and Lerman, highlight the pedagogical practices employed by schools that are also important sources for the teaching of values, as well as the messages that teachers convey concerning respect and responsibility through their teaching methods (Burton, 1995; Murphy, 1996). An approach commonly used by teachers across different subject areas to encourage communication and the sharing of ideas and respect is group work. But group work is viewed suspiciously by many teachers, partly because of the perceived loss of control they experience when they give students opportunities to talk with each other (Doyle, 1983). Teachers are also reluctant to employ group work as they have found that groups do not always work well together (Slavin, 1990). A common problem in the enactment of group work is an uneven distribution of work and responsibility among students, with some students doing more of the work and others choosing to opt out or being forced out of discussions. Cohen and Lotan (1997) developed an approach to make group work more effective and more equitable called 'complex instruction'. This is a teaching method that is specifically designed to counter social and academic status differences in groups, starting from the premise that status differences do not emerge because of particular students but because of group interactions. The complex instruction approach has a number of strands that pertain to the learning of respect and responsibility. In the first instance, the authors recommend that classrooms need to be 'multidimensional' (Rosenholtz & Wilson, 1980; Simpson, 1981). According to the guidelines, unidimensional classrooms are those in which only some practices are

valued; a unidimensional mathematics classroom, for example, would be one in which students are valued for executing procedures and nothing more. Multidimensional classrooms expand the dimensions along which students are judged and encouraged. For example, a multidimensional mathematics classroom could reward students for using different methods, asking questions, representing ideas and having good discussions in addition to the execution of procedures. The theory is that as classrooms become more multidimensional more students have access to ideas and may be regarded as contributing in important ways. When classrooms are multidimensional the authors propose that teachers then apply a 'multiple ability treatment'. This involves explaining to students that no one student will be 'good on all these abilities' and that each student will be 'good on at least one' (Cohen & Lotan, 1977, p. 78). Another feature of the complex instruction approach is a teacher practice that the authors refer to as 'assigning competence'. This is a practice that involves teachers raising the status of students that may be of a lower status in a group, by, for example, praising something they have said or done that has intellectual value, and bringing it to the group's attention; asking a student to present an idea; or publicly praising a student's work in a whole-class setting. Cohen (1994) recommends that if student feedback is to address status issues, it must be public, intellectual, specific and relevant to the group task (Cohen, 1994, p. 132). The public dimension is important as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work; and the specific dimension means that students know exactly what the teacher is praising. The critical and unusual characteristic of 'assigning competence' is the focused attention on particular students who are positioned as low status in their groups. Another feature of the complex instruction approach is the use of roles, with students being asked to play such roles as 'facilitator', 'team captain', 'recorder/reporter' or 'resource manager' (Cohen & Lotan, 1997). The premise behind this idea is that all students have important work to do in their groups, without which the group cannot function.

Various researchers have studied complex instruction in action and reported important learning gains for students (Bower, 1997; Neves, 1997; Ben-Ari, 1997). But although the approach focuses upon the ways in which students treat each other and the respect they employ as they consider each other's ideas, few researchers have considered the implications of complex instruction for equity or for the values students learn through their interactions with others. This may reflect a broader phenomenon that Halstead and Taylor (2000) have recognized as scholars under-researching cultural diversity issues in areas of school practice (2000, pp. 60–61). In this article I will report upon the implications of complex instruction for equity and for the learning of values such as respect and responsibility for others, including those from a different culture, sex and/or social class.

Whether the teaching of respect and responsibility is employed through a curriculum area such as citizenship education or a pedagogical approach such as group work, there is growing awareness of the need to teach students about respectful human relations, especially across different cultural groups and genders.

As globalization takes hold on the world and the movement of 'people, goods or ideas among countries and regions accelerates' (Coatsworth, 2004, p. 38), scholars are increasingly recognizing the need for students to learn to work collaboratively with others, especially those from different cultures and backgrounds (Suárez-Orozco & Qin-Hilliard, 2004; Gardner, 2004). Interestingly, scholars who consider the educational implications of globalization accompany the principles of communication and respect with certain intellectual capacities such as critical thinking, problem solving and creative analysis (Gardner, 2004), melding the intellectual and the social in ways similar to the authors of complex instruction. Research on the brain adds support to the idea that social and intellectual issues be addressed in tandem as previous divisions of emotion and reason are no longer believed to be tenable in our understandings of brain functioning (Damasio, 1994). The notion of relational equity that I propose in this article and consider through a study of a particular teaching approach also merges the social and the cognitive in its consideration of the intellectual relations that students develop in school.

Relational equity

In studying equity most researchers look for reductions in achievement differences for students of different ethnic and cultural groups and genders, as well as equitable treatment in schools. Post (2004) refers to these two versions of equity as equal outcomes and equal access. Fennema describes three versions of equity as: equal opportunity, treatment, and outcome (1990, p. 2), all of which refer to students' experiences and achievements in school. Gutiérrez (2002) offers a helpful definition of equity as: 'erasure of the ability to predict students' mathematics achievement and participation based solely on characteristics such as race, class, ethnicity, sex, beliefs and creeds, and proficiency in the dominant language' (2002, p. 9) This definition is comprehensive but concentrates, as do others, on students' achievements in school. The version of equity that I propose is closer to Elizabeth Anderson's conception (1999), which she refers to as 'democratic equity'. Anderson is a philosopher and her concern is not with test scores or other measures of educational achievement but with an individual's standing in society. 'Democratic equality is identified by an individual "standing as an equal over the course of an entire life"' (Anderson, 1999, p. 319). This conception is closest to the version of equity I propose as it concerns relations between people and it shifts the focus away from measures of achievement and on to ways of acting between people. In classrooms that promote relational equity students would learn to respect each other's differences, to listen to others who have a different opinion, perspective or experience and to act in equitable ways. This is different from the 'social justice' perspective put forward by Frankenstein, Gutstein and others, who consider the ways mathematics knowledge may be used to combat issues of injustice (Frankenstein, 1989, 1990; Gutstein et al., 1997). In offering the term 'relational equity' I am proposing a different focus, no more or less important than the focus on achievement, that draws attention to the ways students learn to treat each other and the respect they learn for people from different

circumstances to their own. There are many ways in which such equitable relations may be encouraged and formed in classrooms; the version of relational equity that we observed in our study, and that I will describe in this article, had three important strands:

1. respect for other people's ideas, leading to positive intellectual relations;
2. commitment to the learning of others; and
3. learned methods of communication and support.

I will argue in this article that these three qualities are important for the production of citizens (MacIntyre, 1984; Shweder, 2003) and that they may be learned through the teaching of a range of subjects. The ways students learn respect for other people are rarely considered by educational researchers, and when they are, they are generally separated from issues of mathematics, or other subject teaching. But if adults are to live and work in a pluralist society and to participate in society in productive ways, then it seems important that such ways of working and interacting should be taught, and modelled in our schools, and they cannot be isolated within non-academic subject classrooms. The classrooms at Railside school, the focus of this article, were more equitable not only because they produced more equitable test outcomes, but because they produced more equity-minded students, who engaged in positive intellectual relations and treated each other with noteworthy degrees of respect.

Description of the study

In order to consider the impact of different teaching approaches upon students' understanding of mathematics, a team of graduate students and I conducted a longitudinal four-year study of three high schools (ages 14–18). The study involved monitoring approximately 700 students as they progressed through four years of three schools that were chosen because they offered different mathematics teaching approaches. We did not go into the study intending to consider students' learning of respect and responsibility for others, but this theme emerged as we worked in the schools over the four years. The school that is the focus of this article was given the pseudonym of 'Railside', as it was located next to the railway tracks in an urban setting in California. The students were from diverse ethnic and cultural groups and were largely from low-income homes. The other two schools, which we called 'Greendale' and 'Hilltop', were in more suburban settings with less ethnic diversity. Greendale school had little ethnic diversity and almost all students were white; at Hilltop school most students were white or Hispanic. Table 1 gives details of the three schools and their populations of students.

   In order to monitor and analyse the teaching approaches in the three schools we observed over 600 hundred hours of lessons, many of which were videotaped. These lessons were analysed in different ways, as set out below. In addition, we interviewed students in every year of the study to consider their reported experiences and interpretations of mathematics class. Students were typically interviewed in

Table 1. Schools and students

	Railside	Hilltop	Greendale
Approx. number of students enrolled in school	1500	1900	1200
Demographics	38% Hispanic 23% African American 20% White 16% Asian/Pacific Islanders 3% other ethnicities	57% White 38% Hispanic 5% other ethnicities	90% White 5% Hispanic 5% other ethnicities
ELL students	25%	24%	0%
Free/reduced lunch	31%	23%	9%
Parent education, % college graduates	23%	33%	37%

same-sex pairs and we interviewed approximately 60 students each year, sampling high and low achievers from each approach in every school, taking care to interview students from different cultural and ethnic groups. In addition to interviews, we also administered questionnaires to all the students in the focus cohorts in years 1, 2 and 3 of the study (when most students were required to take mathematics). The questionnaires combined closed, Likert response questions, with more open questions that we analysed and coded. The questionnaires asked students about their experiences in class, their enjoyment of mathematics, their perceptions about the nature of mathematics, learning, and students. The observations, interviews and questionnaires combined to give us information on the teaching and learning practices in the different approaches and students' responses to them. Teachers from each approach were also interviewed at various points in the study, although the teachers' perspectives on their teaching were not a major part of our analyses. In addition to monitoring the students' experiences of the mathematics teaching and learning, we also assessed their understanding in a range of different ways, administering tests to all students in the different school approaches as well as longer applied assessments that were given to focus groups and videotaped (Fiori & Boaler, 2004).

Data analysis

Data from our classroom observations were analysed in three different ways. First, we drew upon our observations from class visits and videotapes to produce 'thick descriptions' (Geertz, 2000) of the teaching and learning in the different classes. We collectively watched over 600 hours of lessons, and different observers of the classes and the videotapes discussed and highlighted the most salient features of each approach and the differences between them. Second, we conducted a quantitative analysis of time spent in classes. This involved spending a year observing videotapes and deciding upon the different, mutually exclusive ways in which students spent

time in class. These included such categories as teacher talking, teacher questioning whole class, students working alone, and students working in groups. When agreement was reached on the categories that should be used, three researchers coded lessons until over 85% agreement was reached on the coding. We then completed the coding of over 55 hours of lessons, coding every 30-second period of time. This yielded 6800 coded segments. We also recorded the amount of time that was spent on each mathematics problem in class. This coding exercise was only performed on year one classes as it was extremely time intensive and we lacked the resources to perform the same analysis every year, but qualitative observations of lessons suggested that similar differences between the different approaches pertained to each year of the study. In addition to these qualitative and quantitative analyses of lessons, we performed a detailed analysis of the questions teachers asked students. This level of analysis fell between the qualitative and quantitative methods we had used and was designed in response to our awareness that the teachers' questions were an important indicator of the mathematics on which students and teachers worked (see Boaler & Brodie, 2004). Our coding of teacher questions was more detailed and interpretive than our coding of instructional time but it was sufficiently quantitative to enable comparisons across classes.

In order to analyse the detailed student interviews they were first read by teams of researchers and then coded (Glaser & Strauss, 1967; Miles & Huberman, 1994). Codes were first identified by different researchers using a process of open coding; the list of agreed-upon codes was then used to recode all interviews. Questionnaires were analysed quantitatively, with both individual questions and scales of questions being subject to exploratory and confirmatory statistics. The assessments in our study ranged from tests that were scored, blind, and statistically analysed, to problem-solving sessions that were videotaped and assessed. For the videotaped assessments individual student work was graded and rubrics were developed and used to assess the interactions of groups as they worked (Fiori & Boaler, 2004). In addition to the individual analysis of each data source (lesson observations, interviews, videos, questionnaires, assessments) the findings from these multiple sources were then analysed and understood in relation to one another, thus illuminating trends and themes across sources and affording the opportunity to triangulate the data. As data were analysed by a team of researchers, themes were discussed and agreed upon by groups of people, which served to increase confidence in our analyses and findings (Eisenhart, 2002). In addition, constant comparison across cases (Glaser & Strauss, 1967, 1991) was used to illuminate critical defining features and practices of each school. This allowed us to capture subtle aspects of each learning environment that may have otherwise been overlooked. The analyses were shared with the teachers as a form of *member check* (Glesne & Peshkin, 1992), further enhancing the validity of the findings. The three main themes in this article, of valuing different viewpoints, student commitment, and methods of communication and support, emerged from the observations of classes, the student interviews, and the questionnaires and they will be the main data sources reported in this article.

Results

The majority of students in our study experienced one of two teaching approaches. The two suburban schools, Greendale and Hilltop, offered a choice between a 'traditional' sequence of courses taught traditionally and a 'reform' sequence taught using more open problems and group work, but few students chose the reform courses and we had insufficient numbers to include in our analyses. Most of the students at Greendale and Hilltop therefore experienced a traditional approach, as named by the schools. The teachers lectured and the students practised methods, working their way through short questions. Our coding of videotapes allowed us to categorize the ways in which students spent time in their classes. This showed that approximately 21% of the time in 'traditional' algebra classes at Greendale and Hilltop was spent with teachers lecturing, usually demonstrating methods. Approximately 15% of the time, teachers questioned students in a whole-class format. Approximately 48% of the time, students were practising methods in their books, working individually; approximately 11% of the time they worked in groups, and students presented work for approximately 0.2% of the time. The average time spent on each mathematics question was 2.5 minutes.

The second major approach in our study was the approach offered at Railside school in which the teachers posed longer, conceptual problems; students worked in groups and they often presented their work while teachers questioned presenters and other students. Our coding of videos showed that teachers lectured to classes for approximately 4% of the time. Approximately 9% of the time, teachers questioned students in a whole-class format. Approximately 72% of the time the students worked in groups while teachers circulated the room teaching methods and asking the students questions of their work, and students presented work for approximately 9% of the time. The average time spent on each mathematics problem at Railside was 5.7 minutes. An additional important difference between the schools was that Greendale and Hilltop employed ability grouping and students were placed into one of three different levels of classes at the beginning of high school. At Railside all students were placed into heterogeneous algebra classes.

The students at Railside started high school at significantly lower mathematics levels than the students in the more suburban schools ($t=-9.141$, $p<.001$, $n=658$), but within two years they were outperforming the other students, scoring at significantly higher levels on mathematics tests ($t=-8.304$, $p<.001$, $n=512$). By year 4 41% of Railside seniors were in advanced classes of pre-calculus or calculus (similar to one year of A-level mathematics) compared to approximately 27% of seniors in the other two schools. In addition, questionnaire and interview results each year showed that the Railside students were enjoying mathematics more than students at the other schools. Railside teachers were also extremely successful at reducing the achievement differences between groups of students belonging to different ethnic groups at the school. At the beginning of high school Asian students (predominantly East Asian), Filipino and White students were each outperforming Hispanic and Black students. At the end of Year 1, only one year after the students started at Railside, there were no longer significant differences between the

achievement of White and Hispanic students, nor Filipino students and Hispanic and Black students. In subsequent years the only consistent difference that remained was the high performance of (East) Asian students, who continued to significantly outperform Black and Hispanic students, but differences between White, Black and Hispanic students disappeared. Achievement differences between students of different ethnicities at Hilltop, where approximately half were White and half Hispanic, remained, with the White students outperforming Hispanic students, reflecting inequities that are fairly typical for urban schools in the USA (Haberman, 1991; Kozol, 1992). At no time in the study were there any achievement differences by gender in any of the schools and girls and boys were represented equally in the different classes.

Relational equity

The mathematics learning of the students at Railside has been analysed in separate papers (Boaler & Staples, forthcoming). This article will focus upon the unusual values and equitable relations that students developed that are not often the focus of research analyses. Although these were not intended as a focus of our study, it was not possible to spend years in the classrooms at Railside without noticing that the students were learning to treat each other in more respectful ways than is typically seen in schools. In interviews with seniors at the end of high school they told us that the ethnic cliques that were evident in other schools did not form at their school because of the mathematics approach used at the school. Some students explicitly contrasted mathematics classes with other classes in which they had sat in groups but they did not learn to respect, or even know each other. The following analysis will report upon three particular characteristics that students learned and that I argue are important for the achievement of relational equity. These were themes that emerged from our long-term observations and from student interviews. None of these are specific to mathematics and they could be encouraged in any classroom, even though the particular ways that the discipline was introduced and experienced played an important part in the opportunities afforded.

1. Commitment to the learning of others

A major part of the equitable results attained at Railside was the serious way in which students were taught to be responsible for each other's learning. Many schools employ group work which, by its nature, brings with it an element of shared responsibility; but Railside teachers went beyond this to ensure that students took the responsibility very seriously. This had an important by-product; not only did the shared responsibility increase opportunities for students' learning of mathematics, as the teachers hoped, but it taught students to take responsibility for each other and to regard that responsibility as an important part of life. Two themes emerged from the interviews with students that reflected the increased responsibility they learned for each other. One theme reflected the concern students developed for each other's learning. The other reflected the actions they took when other students were not working.

1. Reciprocity. In previous research studies in England and the USA I have interviewed many hundreds of students who have worked in groups (Boaler, 1997a, 2000). In virtually all cases students have reported that they prefer to work in groups than to work alone, but the students in all the other schools in which I have researched have listed benefits that were exclusively about their own learning. At Railside students also talked about the value group work added to their learning, but students' descriptions were distinctly reciprocal and they voiced a clear concern for the learning of their classmates. For example:

> Int: do you prefer to work alone or in groups?
>
> A: I think it'd be in groups, 'Cause I want, like people that doesn't know how to understand it I want to help them. And I want to, I want them to be good at it. And I want them to understand how to do the math that we do. (Amado, Railside, Y1)

The students at Railside did not only learn to help each other and to engage each other in work, they came to enjoy and value such practices:

> It's good working in groups because everybody else in the group can learn with you, so if someone doesn't understand—like if I don't understand but the other person does understand they can explain it to me, or vice versa, and I think it's cool. (Ana & Latisha, Railside, Y3)
>
> Like he was saying, I like the feeling of helping people and the feeling that you get when someone else knows something and picks up on a problem or an answer and then they're able to apply it and help themselves out. It kind of makes you feel like you're helping along the teacher. And I like that. (Jon, Railside, Y4)

Students learned to value the act of helping and to care about the learning of other students. In interviews students told us that they learned this from their mathematics classes and contrasted such classes with others at the school in which they worked in groups but did not learn to appreciate other students or want to help them.

2. Responsibility when things go wrong. In order to understand the nature and extent of the responsibility students learned for each other we presented 78 seniors, 39 from Railside and 39 from the traditional approaches, with different scenarios in interviews. One question we asked students was what they did when other students needed help and they did not understand. The students from the traditional classes did not seem different from the Railside students in their indications of concern for those who did not understand. But a statistical difference in the extent of their responsibility emerged when we asked about students who were less motivated. In particular, we asked students whether they perceived unmotivated students as a responsibility or a burden. We categorized students' responses to this question on a 3-point scale, with 1 denoting low levels of responsibility and an individual orientation, with such comments as:

> Yeah, I never really took my time with those kids, I just said 'alright, go talk to Mahey [teacher] or to Mr Cross [head of department]. I don't have time for this, I've got to get this done so I can get my grade. (Ben, traditional approach, Greendale, Y4)

Comments were given a 2 if they indicated some responsibility, and they were given a 3 if they indicated a high level of responsibility and clear commitment to other students, for example, Neil's response to the same question about unmotivated students:

> I feel it was a responsibility because if you know something you have to, and somebody doesn't know, I feel it's your right, it's, you have to teach them how to do it. Because it's only fair to them that they get the most out of it as you're getting out of it because you're both in the same classroom. (Neil, Railside, Y4)

These different categories, reflecting the extent of responsibility students learned when difficulties arose revealed a statistical difference between the cohorts, as can be seen in Table 2.

Fifty-nine per cent of the Railside students communicated high levels of responsibility towards their peers compared to 5% of the students taught traditionally, a statistically significant difference ($\chi^2 = 2.3125$, $p < .001$, df=2). This result was not surprising; we had observed hundreds of classes and seen the productive ways that students worked together at Railside as well as the serious ways students took responsibility for others. One of the most impressive indicators of the responsibility at Railside was students' persistence when they were given work to do. Even when one person would go off task and chat about non-work issues, others would bring the group back to the work at hand. Instead of viewing uncooperative students as a burden, students regarded them as a responsibility. We did not observe such levels of persistence and motivation in the traditional classes, as two students from those classes describe:

> J: What would happen if someone in your group just didn't want to work?
>
> B: Usually we'd all kind of lose focus.
>
> J: Really?
>
> B: Yeah. Sometimes it seemed like one person kind of went off, and if we were all disinterested enough we'd all kind of drift off away from the subject too. (Jesus, traditional approach, Hilltop, Y4)

Differences in the motivations of groups partly reflected the students' levels of interest in the work, which were higher at Railside, but they also reflected the careful ways the teachers at Railside taught students to work together.

The Railside students came to view each other more respectfully partly because the teachers worked hard to create classrooms in which learning was seen as a collective rather than an individual endeavour. This involved teaching students to be responsible for each other's learning, something that would be perceived as

Table 2. Percentage of students indicating low, medium and high levels of responsibility

	Low	Medium	high	n
Traditional approach	57	38	5	37
Railside approach	15	26	59	39

controversial, or negative in some circles. Teachers encouraged this in different ways, including giving a constant message that students needed to work together as a group and that they needed to make sure all members of a group understood the work. They also reinforced the message by grading the discussions taking place in a group, and occasionally giving group tests in which students worked through a test together, but the teachers graded only one of the individual papers and that grade stood as the grade for all the students in the group. The students learned that mathematics at Railside was a collective phenomenon:

> Math is really about group work 'cause you have like group tests and everything, so if you don't get it then you have to depend on your group a lot (Sue, Railside, Y4)

> It's kind of the rest of your group's responsibility to make them understand. You know, you've got at most three people there to help you understand. (Jose, Railside, Y4)

An additional important way in which responsibility was encouraged was through a practice of asking one student in a group to answer a follow-up question after a group had worked on something. If the student could not answer the question the teacher would leave and come back. In the intervening time it was the group's responsibility to help the student learn the mathematics they needed to answer the question. The practice of asking one member of a group to give an answer and an explanation, without help from their group-mates, was a subtle practice that had major implications for the classroom environment—it meant that students always knew that they must take responsibility for others. In the following interview extract the students talk about this particular practice:

> Int: Is learning math an individual or a social thing?

> G: It's like both, because if you get it, then you have to explain it to everyone else. And then sometimes you just might have a group problem and we all have to get it. So I guess both.

> B: I think both—because individually you have to know the stuff yourself so that you can help others in your group work and stuff like that. You have to know it so you can explain it to them. Because you never know which one of the four people she's going to pick. And it depends on that one person that she picks to get the right answer. (Gisella & Bianca, Railside, Y2)

The students in the extract above make the explicit link between teachers asking any group member to answer a question, and being responsible for their group members. They also communicate a particular social orientation that became instantiated through the mathematics approach, saying that the purpose in knowing individually is not to be better than others but so 'you can help others in your group'. At the end of the first year of our project we heard some resistance from the higher attaining students, who complained about having to spend a lot of time explaining work to others, but by the end of the second year they had changed their minds, as the following students explain. The two seniors (year 4) were asked whether they regarded the need to help others as a responsibility or a burden; they replied:

> I think people look at it as a responsibility, I think it's something they've grown to do like since we've taken so many math classes. So maybe in ninth grade it's like Oh my

> God I don't feel like helping them, I just wanna get my work done, why do we have to take a group test? But once you get to AP Calc you're like, Ooh I need a group test before I take a test. So like the more math you take and the more you learn you grow to appreciate, like, Oh Thank God I'm in a group! (Imelda, Railside, Y4)

The students changed their minds partly because they appreciated the ways the act of explaining work deepened their own understanding and partly because their orientations had shifted from regarding their enterprise as individual and competitive to regarding their work as that of a collective. The sentiment expressed by the following boy also reflects the ways in which students learned to regard themselves as part of a collective working together:

> Because when you kind of think about it, you're only as strong as the weakest … not really the 'weakest' but the person who doesn't really understand it the most. (Jon, Railside, Y4)

The actions of the teachers to change mathematics to a collective endeavour and to keep all students working together, even if some were able to move faster than others, is a contentious practice that many would regard as inappropriate. In particular, people often worry about the learning opportunities for high attainers who spend their time helping others. But our statistical analyses showed that the students who entered Railside at the highest levels attained more than the high-attaining students at the other schools even though the other high-attaining students went into tracked classes and worked with students of a more similar attainment level. Indeed, the higher attaining students were probably the most well served by the Railside approach as analyses of the students' learning trajectories showed that their learning accelerated more than any other students.

The fact that the heterogeneous groups worked so well together and that no one was left struggling or feeling bored was due to the careful work of the teachers. The teachers created a collective and hardworking enterprise by establishing and reinforcing the message that everybody had different strengths, and that there were many ways to be 'smart'. If they found that one student was standing out by, for example, being faster in their mathematical thinking, they would find aspects of the work that the student was less good at and for which they needed more practice—asking them to give more care to their explanations or to helping others understand, for example. This worked to benefit the student who was racing ahead as well as communicating an overall message—that everyone is important and that people have different strengths. Such messages were heard and believed by students, as evidenced by their comments about peers in interviews and by questionnaire responses. For example, in one of our questionnaires we offered the statement, 'Anyone can be really good at math if they try'. At Railside, 84% of the students agreed with this, compared with 52% of students in the traditional classes ($n=473$, $t=-8.272$; df=451; $p<.001$).

In many mathematics classrooms, including the traditional classes of Greendale and Hilltop, students work in groups but learning is an individual and competitive endeavour. Even if teachers do not stress a competitive approach students often measure their own success by the success of others and only know they are doing

well if they are doing better than others. The following student from the traditional classes told us that she had started to doubt her ability in mathematics because of people who 'are better':

> I don't know, I've always been I guess good at math—up until like maybe the last 2 years, realizing … Oh, I'm not really the … there's people that are better at math than I am, you know? (Sarah, traditional approach, Y4, Greendale)

The students who learned traditionally only knew they were doing well if they were doing better than others. This appeared to have an impact on the ways they thought about each other, and students from the traditional classes started to classify themselves and others into categories, such as 'smart' and 'dumb' in their mathematics classes. In interviews we found that students from the traditional classes used an evaluative discourse when describing other students. The following student uses language and sentiments that we did not hear from the students at Railside:

> I don't want to feel like a retard. Like if someone asks me the most basic question and I can't do it. I don't want to feel dumb. And I can't stand stupid people either. Because that's one of the things that annoy me. Like stupid people. And I don't want to be a stupid person. (Scott, traditional approach, Y1, Greendale)

We do not know how pervasive such ideas were from the students in the other schools as we were not looking for them but they were noticeably absent at Railside. This did not mean that Railside students were unaware that students differed in their intellectual contributions; they were equally aware of the diversity of classes but they, like their teachers, came to appreciate the diversity in their classes, as one student reflected:

> Everybody in there is at a different level. But what makes the class good is that everybody's at different levels so everybody's constantly teaching each other and helping each other out. (Zane, Railside, Y2)

Such sentiments reflect a very careful implementation of a mixed-ability approach, one in which teachers strove to expand the dimensions along which students regarded each other.

2. Respect for other people's ideas

Another critical aspect of relational equity that emerged from our data and that enabled the supportive relations that students developed was the respect that they learned for different ideas and viewpoints, a goal that is explicitly articulated in the National Curriculum Citizenship documents. This respect was particularly important because students came from diverse backgrounds and cultural groups. The students learned that different mathematical ideas were useful and they learned to value the different people giving such ideas. This helped the students to appreciate the diversity of people around them. For example, two students who told us they liked maths gave the following reasons:

> T: You got everyone's perspective on it, 'cause like when you're debating it, a rule or a method you get someone else's perspective of what they think instead of just going off your own thoughts. That's why it was good with like a lot of people.

C: I liked it too. Most people opened up their ideas. (Tanita & Carol, Railside, Y4)

Many students talked about the ways they learned to have open minds and to respect different ways of thinking:

Int: What do you guys think it takes to be successful in math?

A: Being able to work with other people.

E: Be open minded, listen to everybody's ideas.

A: You have to hear other people's opinions 'cause you might be wrong.

E: You might be wrong 'cause there's lots of different ways to work everything out.

A: 'Cause everyone has a different way of doing things, you can always find different ways to work something out, to find something out.

E: Someone always comes up with a way to do it, we're always like 'Oh my gosh, I can't believe you would think of something like that.' (Ayana & Estelle, Railside, Y4)

Additionally, as students learned that different interpretations and perspectives on mathematics problems were important, they also learned that different interpretations on issues and problems in life were helpful. Richard Shweder (2003) talks about the importance of considering different perspectives on issues in the working of a democratic society:

It is often advantageous to have more than one discourse for interpreting a situation or solving a problem. Not only alternate solutions but multidimensional ones addressing 'several orders of reality' or 'orders of experience' may be more practical for solving complex human problems. (Shweder, 2003, p. 100)

The Railside students appeared to develop the perspectives Shweder describes in mathematics, as the act of considering different methods and perspectives on complex mathematical problems encouraged students to see the value of different perspectives more generally. The following student reflects on the act of considering different mathematical methods:

I think it helps, because it helps with learning to get out of your comfort zone, 'cause whenever you learn, you're not always going to learn the exact way, so to be able to learn different types of ways, if someone interprets something the way they do, and then you look at it and you're like: 'oh look at this', and you see it their ways, you never know later on when you might have to change your interpretation or something. So it allows you to come out of like your comfort zone. (Ayana, Railside, Y4)

This student talks naturally about the importance of valuing other people's perspectives and interpretations as she discusses mathematical methods, reflecting the important connection between mathematics and social relations that I hope to convey in the notion of relational equity. The following student also talks about the ways of thinking and working that people learned more generally:

Int: And you were saying that the approach has been helpful for you in life?

J: Yeah, 'cause it allows you to think like beyond the obvious, not just what's in front of you but look past that. (Jessica, Railside, Y4)

The students at Railside reported in interviews that they learned to respect and learn from students of different cultures because of the interactions in mathematics class:

> R: I love this school, you know? There are schools that are within a mile of us that are completely different—they're broken up into their race cliques and things like that. And at this school everyone's accepted as a person, and they're not looked at by the color of their skin.
>
> Int: Does the math approach help that or is it a whole school influence?
>
> J: The groups in math help to bring kids together.
>
> R: Yeah. When you switch groups that helps you to mingle with more people than if you're just sitting in a set seating chart where you're only exposed to the people that are sitting around you, and you don't know the people on the other side of the room. In math you have to talk, you have to voice if you don't know or voice what you're learning. (Robert & Jon, Railside, Y4)

The act of considering different students' ideas on mathematical problems helped the students appreciate the contributions that others could bring to the solution of problems. This appreciation was enhanced by the students' heightened awareness of the different ways in which a student may contribute mathematically, deriving from the multidimensionality of the classrooms. Indeed, the positive intellectual relations that were formed between students were enabled, in the first instance, by the multidimensionality of classrooms. At Railside the teachers created multidimensional classes by valuing many dimensions of mathematical work. For example, the students were given fairly open problems that they could solve in different ways, and the teachers valued different methods and solution paths, which enabled more students to contribute ideas and feel valued. But multiple solution paths were not the only contributions that were valued by teachers. When we interviewed the students and asked them, 'what does it take to be successful in mathematics class?', they offered many different practices, such as:

- asking good questions;
- rephrasing problems;
- explaining well;
- being logical;
- justifying work;
- considering answers; and
- using manipulatives.

Students from the 'traditional' classes described the valued practices in mathematics class in much more narrow ways, saying only that they needed to concentrate, and pay careful attention. The multidimensional nature of the classes at Railside was an important part of the increased success of students—because there were more ways to be successful, many more students were successful. Students developed an awareness of the different practices that were valued and they felt successful because they were able to excel at some of them. Teachers at other schools may not encourage practices outside of procedure execution because they are not needed in national examinations, but the fact that teachers at Railside valued a range of

practices and more students could be successful in class made students feel more confident and positive about mathematics. This probably enhanced their success on tests even when tests assessed a narrower range of mathematical work.

The following comments given by students in interviews give a clear indication of the multidimensionality of classes:

> Back in middle school the only thing you worked on was your math skills. But here you work socially and you also try to learn to help people and get help. Like you improve on your social skills, math skills and logic skills. (Janet, Railside, Y1)

> J: With math you have to interact with everybody and talk to them and answer their questions. You can't be just like 'oh here's the book, look at the numbers and figure it out'.

> Int: Why is that different for math?

> J: It's not just one way to do it … It's more interpretive. It's not just one answer. There's more than one way to get it. And then it's like: 'why does it work?' (Jasmine, Railside, Y1)

The students recognized that helping, interpreting and justifying were critically valued practices in their mathematics classes. The multidimensionality of classes did not only mean that more students could feel successful and valued, it also meant that students learned to appreciate the different contributions that students made. As teachers valued students seeing problems in different ways, offering different methods, partial solutions, or different interpretations, the students also came to value these different contributions—and the people offering such ideas. This was particularly important at Railside as the classrooms were multicultural and multilingual. The equitable relationships that Railside students developed were only made possible by a conception of mathematics that valued the contribution of different insights, methods and perspectives in the collective solving of particular problems with particular solutions. This was different to the valuing of, for example, different interpretations of a text in an English class. In mathematics students were generally solving problems with one answer. This meant that they needed to resolve any differences in answers and that it was helpful to understand any differences in method. Whereas in English classes students may have agreed to differ in their interpretations, in mathematics classes at Railside students worked to resolve and understand any differences.

3. Learning methods of communication and support

The third aspect of the equitable relations students achieved concerned the practical ways students learned to help each other, for it cannot be assumed that if students are placed into groups they will know how to support and help each other, even if they are motivated to do so. At Railside, we learned through our observations and through interviews that part of the reason for the success of the groups was the sophisticated forms of help the students developed. These ways of communicating and helping were modelled by the teachers in their careful and consistent interactions with students, and learned by students.

An important aspect of the communication students learned was the asking of good questions. Our analyses of the teachers' questions in the different approaches showed that the teachers at Railside asked more questions that were conceptual and probing, rather than procedural (Boaler & Brodie, 2004). Impressively, the students learned to ask similar helpful questions of each other. The students also learned that it was helpful to question other students rather than simply *tell* other students how they had worked:

> I think the biggest help is just to stop with the problem, stop doing it and kind of step back from it and start asking questions, start asking thought-provoking questions about the problem. (Jon, Railside, Y4)

> OK, if one person doesn't understand, then we have to work together and we're gonna, you know, get them asking a question, that's what we're gonna do in calculus, if somebody doesn't get it, we push them to ask a question to help us to get them past that point where they are. (Latisha, Railside, Y4).

The Railside students learned to ask others to stand back from problems and consider what they were asking—a practice that we observed the teachers enacting on frequent occasions. As the students reflect:

> Like, when I'm in a group I try to make sure that everyone understands it. Like some people just wanna get their work done so they can say Oh, I'm done. But me, I don't like to leave people behind … So like I know if someone's sitting there quiet it's probably because they don't know how to ask a question or they just don't get it so then I'll help, I'll start from a bigger question about what we're really doing. (Ana, Railside, Y4)

> You just start from the beginning, you're like what am I doing? What am I trying to figure out? What do I need to help me figure this out? (Latisha, Railside, Y4).

The students also tried to motivate each other when some students appeared unmotivated:

> I try to get them to think about if you do good in this class, then you can get into college and you can get better grades and you can be more of a successful person rather than … and you can take advantage of being good right now, being more successful later on. 'Cause if you give up right now, you're probably going to give up for a long time 'cause people don't talk to you and turn you around, you don't realize it. (Arnetha, Railside, Y4).

And they were persistent in their attempts to get everybody working:

> I kind of try to push the person-not-working to work. I mean they might not listen to me, but I'll sit there and go … I'll bother them to the point where, if they don't work, then they'll be really annoyed and stuff. (Andrew, Railside, Y4)

The students gave clear explanations of the ways they would approach students who were not working, reflecting a supportive practice that we did not witness at the other schools:

> I just kind of tell them 'well are you going to act serious now? Because you need to!' There's a person—I won't say no names—but who is in our group right now and he just kind of lies around. And it seems the other guy, he just likes doing stuff by himself, and I go over and talk with him and once I'm done talking to him I go over and I sit by the one

> who really doesn't care and I tell him you know—'are you understanding this stuff?'(Caroline, Railside, Y3)

Such levels of responsibility for classmates among high school students do not seem typical. The different methods that students learned to help and motivate each other were both a source of their ultimate high achievement and an indication of the responsibility that they developed for their classmates.

When researchers have considered the beliefs students hold about mathematics they have tended to concentrate upon two separate views of the subject; one that is characterized by a focus on the rules and methods that make up part of the subject, another that also includes the concepts and ideas that connect the different rules and methods (Schoenfeld, 1992). At Railside the students communicated views of the subject that did not fall along these dimensions. Many of the students told us that mathematics was a language, a form of communication and interpretation. They told us that the purpose of knowing mathematics was to express ideas, for example:

> It's just a different form of expression. That's why I like it more. (Shaun, Railside, Y3)

And that the subject itself was a language, as these boys communicate:

> I think math is like kind of a language, because it has got a whole bunch of different meanings to it … and I think it is communicating. When you know the solution to a problem, I mean that is kind of like communicating with your friends. (Jose, Railside, Y3)

> I think of it like a language class kind of. Math seems like a second language or another language that we're learning—because it is something that you can use to communicate to others through math. (Jon, Railside, Y4).

The students' communicative notions reflected both the responsibilities they held for each other and an unusually 'connected' way of thinking about mathematics (Gilligan, 1982; Belencky *et al.*, 1986; Becker, 1995). The students' views were unusual and they came from the integrated social and mathematical system the teachers had created.

Discussion and conclusion

The Labour government has supported ability grouping in schools since it came into power, despite a wealth of research studies that show the ineffectiveness of ability grouping practices (Ball, 1984; Linchevski & Kutscher, 1998; Boaler, 1997 a,b,c); at the same time they have voiced a concern to teach students citizenship skills and to equip students to be fair and responsible members of society. These two goals may be inconsistent. Even if ability grouping had been shown to raise academic standards, which it has not, it does not provide students with the opportunities to respect students who are different from themselves. Indeed, studies have shown that ability grouping polarizes students, keeping apart students of different social classes (Ball, 1984; Boaler, 1997b). This study has shown the ways in which heterogeneous grouping and an associated set of teaching practices allowed students to interact with others from different social classes, cultural groups and ability levels and to broaden

the ways they regarded other students, respecting the diverse ways in which different students approached mathematics work. The ethnic cliques that were evident in other schools in the area were not evident at Railside and the students partly attributed their productive relations with students across cultures to the mixed-ability mathematics approach they experienced.

Critical to the approach at Railside was the multidimensional version of mathematics that the students experienced. The students were required to solve conceptual problems that were chosen because they could be solved in many different ways and students were valued for such practices as asking exploratory questions, representing ideas in different ways, and making connections; practices that are often absent in mathematics classrooms. Tim Brighouse recently summarized the 'great belief' uniting two of England's most influential school reformers, Brian Simon and Caroline Benn, in the following way:

> All children, however diverse, learn best when they learn together, sharing each other's insight and experience, absorbing knowledge and recreating knowledge as they collaborate, in the company of their teachers in a common pursuit. (Brighouse, 2003, p. 3)

Mathematics teachers rarely create the sort of collaborative classroom communities to which Brighouse alludes, believing that mathematics is too hierarchical and closed to be the subject of collaboration and negotiation. The Railside teachers showed that mathematics is as open as any subject to the opportunities for students to learn powerful forms of communication, as well as respectful and positive intellectual relations.

I hope to have made two points about equity in this article. The first is that equity should not only be measured by test scores and that we need to consider the respect and relations that develop between students of different circumstances. The second is that some routes to equity are not found within the content of curriculum. In the USA there is a lot of attention to the notion of cultural modelling (Lee, 2001), premised upon the idea that students will be advantaged if closer connections are drawn between cultural practices in which they engage at home and out of school, and those that are considered and valued in school. This is clearly an important area of research and practice but it is not the *only* way in which equity may be pursued. At Railside teachers did not use a cultural modelling approach but they pursued equitable values and practices through the pedagogy that was employed in mathematics classes. The pedagogical practices that were developed resulted in equitable relations *and* the diminishment of achievement differences between students from different cultural groups. Each of the three aspects of relational equity I have presented, of responsibility, respect, and support, is included in the NCC's citizenship education curriculum (2006), but it is important to note that these values may be taught within mathematics, and at Railside school they combined to produce high and equitable achievement. Indeed, the powerful outcomes achieved at Railside raise questions about the curricular home of some important principles and practices that are often left to un-assessed, low-status subject areas in England and Wales. Relational equity describes a positive and equitable set of intellectual relations

between students, similar to the relations formed in the mathematics classrooms at Railside. But the practices that were taught at Railside did not only impact on the group relations in classrooms as I contend that individuals in the classrooms learned important values and principles that went beyond the ways that they worked in groups. The students learned in their mathematics classrooms that they could solve complex problems through persistence and collaboration with others. They learned to value the different and varied ways in which different people solved problems. They learned to respect students from different ethnicities, genders, and social classes and they learned effective methods of communication. These were extremely valuable lessons for the students to learn and such lessons could remain with them for the rest of their lives.

Some researchers have, importantly, raised questions about the value of student conversations (Sfard & Kieran, 2001), pointing to the careful conditions that teachers need to create in order that students may benefit from such conversations (Boaler & Humphreys, 2005). The teachers at Railside demonstrated one way in which student conversations could be used to aid mathematical understanding and to promote equity. But the conservative educational climate governing British schools at present could make the methods used by the Railside teachers extremely difficult to implement in mathematics classrooms. Mixed-ability teaching is unpopular with policy makers and difficult to enact, but it could be the only method that is viable if students are to 'learn without limits' (Hart *et al.*, 2004). Support needs to be given both to research on ways that mixed-ability teaching can work and to the teachers who enact such approaches. In my recent interviews with 24 year-olds reflecting on their school experiences a student described setting as a 'psychological prison' in his school that diminished students' hopes, aspirations and opportunities (Boaler, 2005); the negative consequences of the setting he described were also shown by many other forms of data (Boaler, 1997a). If British policy makers are serious about promoting citizenship, respect and good behaviour in schools then it may be time to abandon their blind faith in setting (Carvel, 1996; Dixon, 1997) and to acknowledge that equitable relations among students in schools require that equitable classroom conditions be created.

Acknowledgements

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