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Packing Circles
Day 3

Introduction

This activity allows students to explore how to optimize the packaging of circular objects which relates to many real-life scenarios. Students have the opportunity to work collaboratively to determine which packaging strategy is optimal for a set of six discs. Students can think creatively about their packaging strategies and exploring the relationship between the area of circles and that of other shapes.

Agenda

Activity	Time	Description/Prompt	Materials
Mindset Message	10 min	Play the mindset video, ***Speed is not Important***, https://www.youcubed.org/weeks/week-4-grade-9-12/	Mindset Video day 3, ***Speed is not Important***
Explore	25 min	• Introduce the problem. • Give students time to explore Packing Circles.	• Packing Circles Handout • Maths journal • Pencils • Colored pencils or pens • Circular counters • Graph paper • Rulers
Discuss	10 min	Invite students to share their findings and visual proofs.	
Debrief Mindset Message	5 min	Ask students to reflect on the idea discussed in the video that math is NOT about speed. What is important in math is the think care-fully, deeply, and to make connections.	• Maths journal • Pencils

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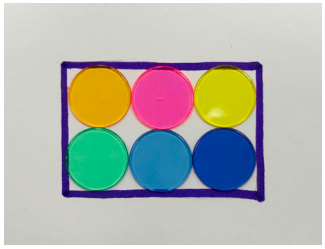
Background

Circle packing is an arrangement of circles that do not overlap and are confined within a boundary. In this task we are asking students to consider the different polygon shapes for packing six congruent circles. For more information about packing circles go to https://en.wikipedia.org/wiki/Circle_packing#Applications_of_circle_packing

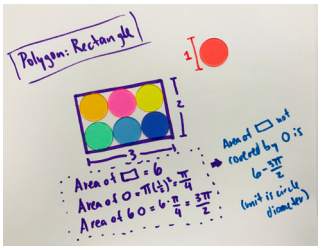
Activity

Introduce the problem to students by showing them six circles of equal size. Pose the question, "What polygon is the optimal package for these six circles?" Mention to students that packing problems are an area of mathematics, involving optimization. These types of problems are often found in real-life, for example in storage and transportation and electoral maps. Share with them additional background information about circle packing included in the "Background" section above or any additional resources.



Emphasize for students they are creating visual proofs to justify the order of their list and what they decide is the optimal package. Allow students to work in partners or groups. Students should describe their polygon, including measures of sides and angles. They should include the area of the polygon and the percent of that area that the circles cover.

Distribute one handout and twelve circular counters per pair of students. Allow students to discuss and decide with their group what polygons are acceptable for packaging. Encourage students to build and make diagrams of the different packages to convince each other of the optimal polygon.



As students are working, notice what strategies they are using to approach this problem. Encourage students to use a variety of strategies. Give space for them to use their own strategies and calculations. This can encourage creativity in students' thinking and highlight that there are many different ways students can approach this problem and mathematics in general.

When pairs or groups think they have finished and decided which polygon package is optimal ask them to share their conjectures and justifications with another pair or group. Encourage them to be skeptical and ask questions about how they know and how they know the other polygons are not optimal. This can inspire students to develop the reasoning and overall strength of their conjecture.

When students have had an opportunity to explore many different shapes for packaging bring the class together to share their findings. Invite students to share different shapes they explored. Encourage

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them to share what they noticed as they tried different polygons. Have students share their ordered list of packages and their visual proofs for how they decided to organize their list. Support students to ask questions of each other and to provide clear explanations for their thinking.

Extension

- What polygon would be the optimal package if you added a seventh circle? What if you kept adding circles is there a polygon that would be optimal for any number of circles?

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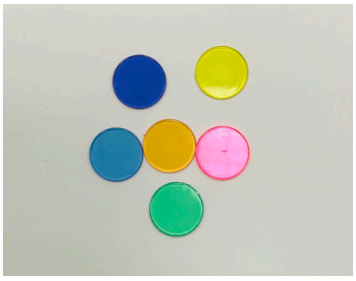
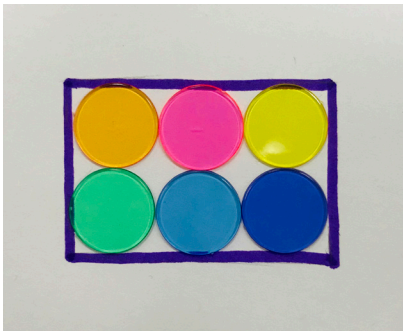
Packing Circles

What polygon is the optimal package for these six discs?



Here is an example of a polygon containing all six discs.



Explore a variety of polygons then organize the polygons in order of optimal packaging of six circular objects. Create a visual proof for how you ordered the polygon packages.

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