



Learning-Disabled Students Make Sense of Mathematics

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Learning-Disabled Students Make Sense of Mathematics

Cal loved mathematics word problems. He delighted in new types of problems, exclaiming, “This is fun!” when a problem was presented. At the end of third grade, he successfully solved word problems involving addition, subtraction, multiplication, division, multiple steps, and extraneous information.

Cal had not always loved mathematics. At the beginning of third grade, he was identified as learning disabled (LD) in language, reading, and mathematics. Although he had strengths in visualizing situations, his weaknesses in memorizing facts and procedures put him at a disadvantage during mathematics instruction.

Evan, a second grader, was identified as LD in mathematics and reading. His classroom teacher stated that he had limited mathematics skills and concepts. Tests revealed that although his verbal reasoning was in the superior range, he had difficulty with mathematical reasoning and computation. When given a mathematics problem, his primary strategy was to guess the answer.

An Instructional Approach That Makes Sense

Some teachers would argue that the best instructional approach for Cal and Evan is to focus on remediating their deficits. Some would recommend teaching the basic facts and computational procedures and providing extensive practice on these skills. Instead, Cal and Evan’s instruction built on their prior knowledge and strengths. This instruction encouraged Cal’s love of mathematics and helped Evan think more and guess less.

Cal, Evan, and three other LD students participated in a study designed to assess and encourage their natural problem-solving strategies (Behrend 1994). Previous research with non-LD children has revealed children’s natural abilities to solve

word problems by modeling the relationships within the problems (Carpenter 1985; Carpenter et al. 1993; Carpenter, Fennema, and Franke 1996). Individual interviews conducted before the eight small-group instructional sessions showed that these LD students also had natural strategies to solve problems, although they did not use them consistently. Instruction revolved around posing a word problem, allowing students time to solve the problem in any way they chose, sharing their strategies, and discussing similarities and differences among the strategies. During the thirty-minute sessions, students usually solved and discussed three to four word problems that represented a variety of mathematical operations. Despite the short duration of the study, individual interviews conducted after the instructional sessions showed that the students were able to solve a variety of word problems by making sense of the problem situation instead of applying a rote procedure.

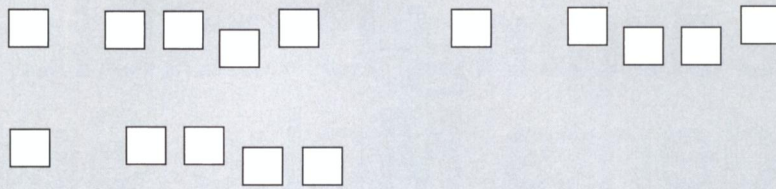
The following examples from Cal’s and Evan’s work highlight this strategy. Because both Cal and Evan had difficulty remembering procedures, instruction that focused on understanding the problem and solving it in a way that made sense

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FIGURE 1

Cal uses counters to represent bags and marbles.



built on their natural problem-solving strategies. Word problems provided a context for the numbers, and students could attach meaning to the numbers and operations.

Examples of Students' Work

Encouraging Evan, who generally guessed the answer, to think about the problem changed his perception of mathematics. He initially believed that problems did not make sense and any answer was as good as any other. Explaining how he solved the problem forced him to think about how he got his answer. During an early session, he was asked to solve this problem: "There are 6 children and 9 cookies. How many more cookies are there than children?" Evan guessed eight. The problem was repeated and Evan was asked to try it again. He said three and explained, "So how I got it is, if each of them had one cookie, and they ate that one cookie, then there would be three left." Evan made sense of the problem by visualizing the situation. Guessing became an occasional strategy rather than a primary strategy as he realized that he could solve the problems. Evan went from solving fewer than half of the problems in the initial interview to solving almost three-fourths of the more difficult problems in the final interview.

Sometimes students needed assistance to make sense of a problem. The following interaction shows how the conditions of the problem were highlighted. The students were given this problem: "There are 3 bags of marbles. Each bag has 4 marbles in it. How many marbles are there in all?" Students solved the problem in their own ways, then shared their strategies.

Cal. OK. I didn't really know how to do it. So I put three down. I had three [holding up three fingers]; then I took, I got four [holding up four more fingers]; then I counted altogether. That makes seven.

Teacher. OK, so you got seven? Does that tell you how many marbles there are? [Cal looks confused.] Would you like me to read the problem again?

Cal. Yeah.

Teacher. Think about the problem. There are three bags of marbles. Got that in your head?

Cal. Yeah, I'm going to need three of these. [He grabs counters.]

Teacher. And there are four marbles in each bag.

Cal. Four marbles in each bag? That's easy.

Teacher. How many marbles are there?

Cal. In all?

Teacher. Yeah, altogether. How many marbles are there?

Cal. Each bag, how many marbles? That would be—there are three bags.

Teacher. Yeah, there are three bags and there are four marbles in each bag.

Cal. There's four marbles. [Cal puts four counters by each of the three "bags" (see fig. 1).]

Teacher. So how many marbles are there?

Cal. Thirteen.

Evan. Twelve.

Cal. What?

Teacher. Do you want to count again?

Cal. Yeah. These are the bags. [He pushes the three "bags" away from the other counters and makes sure that each group has four. He then counts all the counters representing marbles.] Twelve.

Cal did not immediately know how to solve the problem. Being reminded of the conditions of the problem helped him visualize the problem situation and choose a strategy for solving it.

This basic strategy of "understand the problem" allowed students to solve a variety of problems without learning new strategies for different types of problems. For example, in one session Cal successfully solved an addition problem, a subtraction problem involving a comparison, two division problems, and a missing addend problem. Students applied one rule to all problems: "Make sense of the problem."

Because students focused on the problem conditions, they were able to solve problems with multiple steps and extraneous information, even though they were not warned about these new conditions. Cal was presented with the following problem: "Adam has 8 stickers. Annie has 4 stickers. How

FIGURE 2

Evan places two stickers next to each friend to find the number left.

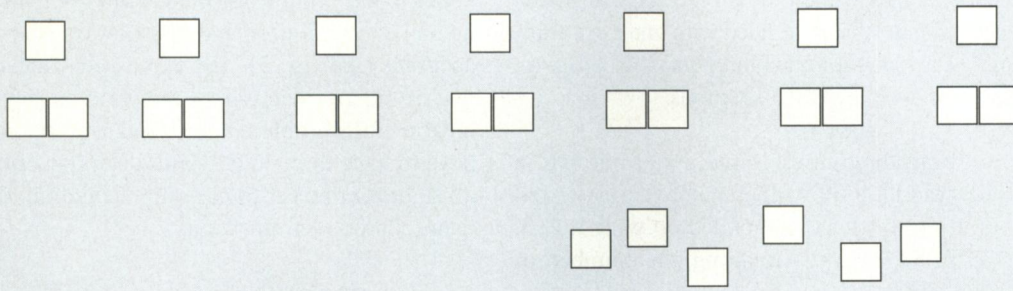


FIGURE 3

Cal correctly determines the number of cars needed.

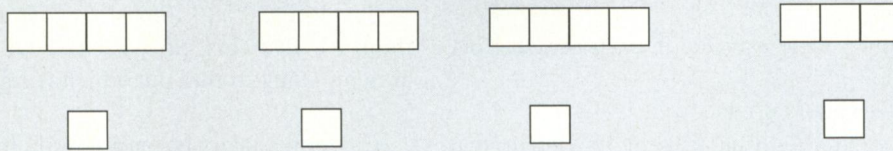
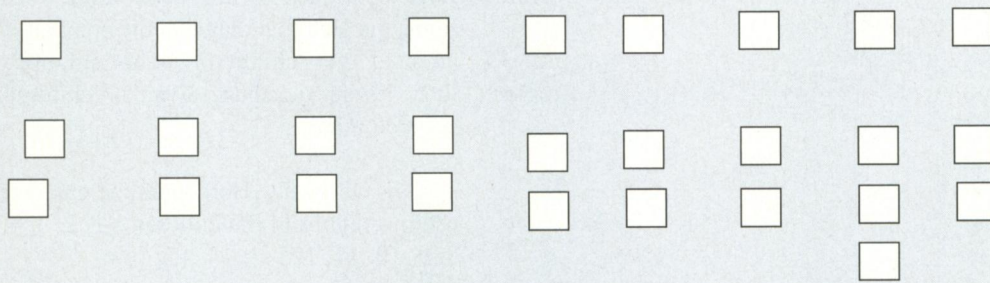


FIGURE 4

Evan puts only two children by each car and uses extra cubes for his solution.



many more stickers does Annie need to get to have 9 stickers?" He laughed and said, "Adam's not in this so I threw him away. . . . And I said four and then I got five. I said 4, 5, 6, 7, 8, 9. I got five." Evan tackled the following problem: "Fran took a walk in the park. She picked 4 flowers. She saw 12 birds. Then she picked 13 more flowers. How many flowers did she pick in all?" Evan began his explanation with "Just forgetting about the birds . . ."

Many of the students' explanations were tied to the problem situation. The students were given this problem: "Amy had 20 stickers. She gave stickers to 7 friends. She gave 2 stickers to each friend. How many stickers did Amy have left for herself?" Evan solved this problem by grabbing two 10-stacks of cubes to represent the twenty stickers. Then he put out seven cubes and said, "These are all the people. Seven people." He started dealing out cubes from the twenty to the seven people, saying, "Now she gives two, and two, and two, and

two—and wait—and two, and two, and one more. So she has 1, 2, 3, 4, 5, 6 for herself" (see fig. 2). Evan understood the problem and needed no assistance to solve it.

Students sometimes got different answers for the same problem. Instead of erasing an answer and changing it to the answer of a more capable student, the students justified their own answers. Through their conversations they made sense of the problems and found their own or others' errors. The following example is from a session with Cal and Evan. The problem was the following: "Fifteen children are going to the circus. If 4 children can ride in each car, how many cars will be needed to get all 15 children to the circus?"

Both Evan and Cal began to model the situation by counting out fifteen cubes to represent the children, using other cubes to represent the cars, and breaking off groups of "children" to go in each car. Cal accurately put four children by each car, until he had only

three left for the last car (see **fig. 3**). Evan put only two children in each car. He became confused and used more cubes for the children than the original fifteen he had taken out. When he ended with one extra cube, he said, “And one will have three in it” (see **fig. 4**). The students explained their solutions.

Evan. I put three in each one. . . . I put two in each one. And I left off with one more extra person, so I put it into this car. And ended up with 1, 2, 3, 4, 5, 6, 7, 8, 9 cars. [Counting the number of groups.]

Teacher. What do you think, Cal?

Cal. I think four, because you said four people can get in a car.

Teacher. I did. I said four people can ride in each car.

Cal. Four people can ride in each car, so I put four people—

Evan. I’ve got to do this over.

Cal. —in that car. I put 1, 2, 3, 4 people in that car. 1, 2, 3, 4 in another car. And I count 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.

In the middle of Cal’s explanation, Evan recognized that Cal’s solution fit the problem structure

of four children in each car and his did not. He then joined each pair of cubes with another pair of cubes to get groups of four. He did not realize that he had extra children and his solution was not yet accurate (see **fig. 5**). He expressed concern that one of the cars had only three children, saying, “I have a little problem here, because one car will have to have three in it.” After deciding that it was all right for one car to have three children, the students shared their answers.

Cal. I got four.

Evan. 1, 2, 3, 4, 5 cars.

Teacher. So, Evan, you think five cars. Cal, you think four cars.

Cal. Yeah.

Teacher. Cal still thinks it’s four cars and you think it’s five cars. So what do you think? How are you going to find out which is right?

Cal immediately began to justify his answer by doing the problem again as he explained what he was doing. Not convinced, Evan was sure that his solution was correct because he had four children in each car, except for the one with three. Cal and Evan continued to talk about each other’s solutions, puzzled that they had different answers. Cal counted the “children” in his solution, fifteen, then began to count Evan’s “children.” Evan counted along.

Cal. Oh, wait. [He counts the cubes that Evan used to represent the children.] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16—

Evan. 17, 18, 19.

Cal. So take this away. [He takes away some of the “children” and counts them (see **fig. 6**).] And you see 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.

Evan. I think he’s right.

Cal. Then I counted them all up and I got fifteen. Then I put all four in each car.

Evan. He’s right. He’s right.

Teacher. Why is he right?

Evan. Because I had too many.

Teacher. You had too many what?

Evan. Too many blocks.

Both students were initially confident in their own answers. During Cal’s explanation, Evan realized that he had missed an important part of the problem: There were four students in each car, not two. He quickly adjusted his response to match this condition, but they still had different answers. Evan and Cal had to analyze the problem and their solutions to discover the reason for that difference. They reviewed the problem until they had met all

FIGURE 5

Evan’s attempt to revise his solution

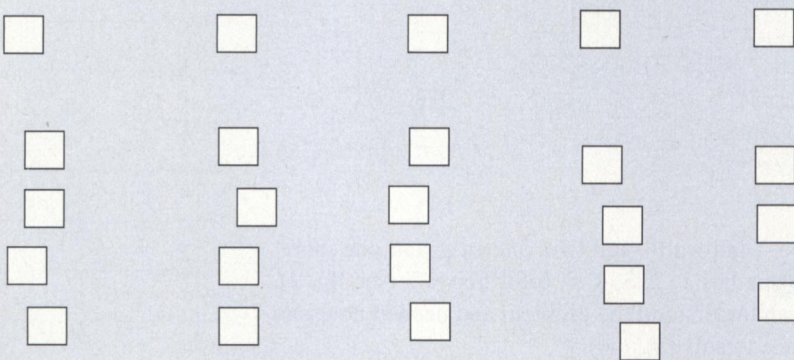
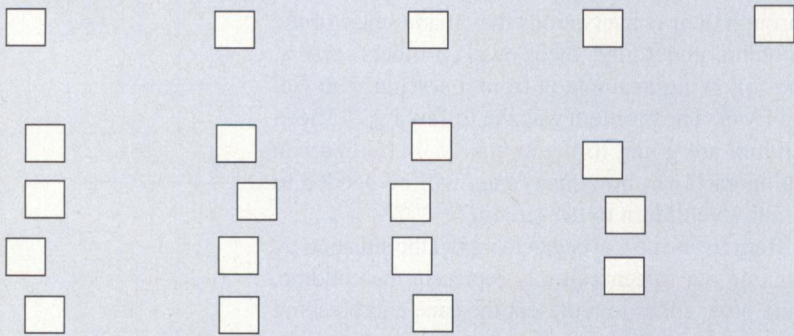


FIGURE 6

Cal corrects Evan’s mistake by removing extra cubes.



conditions. The teacher provided minimal guidance and encouraged the students to make sense of their solutions, which they did.

This instructional approach prepared students to solve many different types of problems, communicate their strategies, justify and explore solutions, and reason mathematically. Because they were able to solve the problems in a way that made sense to them, the students gained confidence in their ability to do mathematics. They willingly spent time solving, checking, and discussing the problems.

Cal loved mathematics, and Evan stopped guessing.

Action Research Idea

How successful are your students in solving problems? To assess their abilities, begin with a small group of students who are having difficulty with mathematics. Gather initial data by asking them to solve several word problems. Include at least one problem with multiple steps or extraneous information. Do not say anything about how to solve the problems or warn them about the multiple steps or extraneous information. You might try the problems in this article or write some of your own. Have the students share their solutions while you keep records of their strategies, and avoid teaching during this initial data collection. Then spend at least a week posing a variety of problems to these students, encouraging them to use their own strategies and to explain their solutions. If they have difficulty, focus on the conditions of the problem, instead of giving clue words or procedures. Listen to their explanations. After these instructional sessions, assess the students' performances on problems similar to the initial data collection. Compare the students' responses. Now how successful are your students in solving problems? Have their explanations changed?

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