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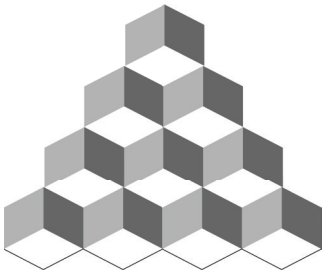
Cantor Ternary Set
Week 4 - Days 3, 4, & 5



Introduction
In this activity, students will explore exponential patterns. They will do so by exploring the Cantor Ternary Set, a mathematical object that has been explored and debated by mathematicians for over a century. Students will come up with their own questions about this curious set, and explore them in depth. They might even have a brush with infinity here or there!

Connection to CCSS
MP1
MP3
MP7
HSA.CED.A.1

Agenda

Activity	Time	Description/Prompt

Launch	30 min	• Looking at the visual representation of the Cantor Ternary Set. How do you see it? Answers can be collected number talk-style.
		• Then the class can brainstorm questions they can come up with about the pattern and its growth. Some examples are:
		  • How many black lines will there be in the 10th row?
		  • What is the total length of the black lines in the nth row?
		  • Will there ever be a row with no black lines?
		  • How many white spaces are there in any row?

Explore 1	20+ min	• Invite the class to break up into small groups to work on the questions they came up with in the brainstorm above.
Explore 2	50+ min	• (OPTIONAL) Introducing the Sierpinsky Carpet and the Menger Sponge as generalizations of the Cantor Ternary Set. • Encourage students to represent the answers they find to each question in several different forms for sharing with the rest of the class later.
Discuss	50+ min	• Invite students to present their questions and answers to the rest of the class, as well as build on each others' ideas.
Reflect	5 min	• How is the growth of this pattern different from the ones you have explored so far?

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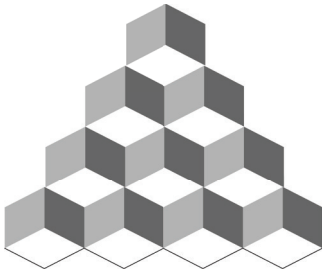


To the Teacher

This activity invites students to explore exponential growth through the Cantor Ternary Set. There are several pieces of it that grow exponentially, such as the amount of black and white segments in each row and the total length of those segments, so we want to give students the chance to come up with their own questions about the set and explore them. This gives students an opportunity to explore a mathematical object, the Cantor Ternary Set, which some mathematicians use in their work. If students are interested, there is a lot more information about the history of Cantor's work and uses of this set online.

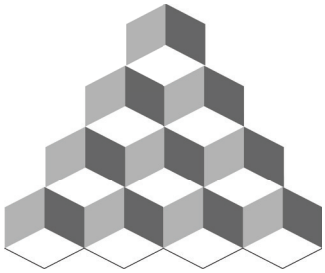
For you to know, but not to tell the students, as they will have an opportunity to discover it: the Cantor Ternary Set is constructed by taking a line segment and removing its middle third (meaning everything between, but not including, the 1/3 point and 2/3 point), and then for every row underneath following the same process for each segment. This might become important if students have questions about precision, but we encourage students to see the pattern in whatever way makes sense to them, even if it's not in the way it was constructed.

Keep in mind that this will be the first time that students have seen and explored an exponential function. Given this is the first time seeing exponential functions, many students will not come up with an exponential expression for this pattern. This is fine. Let students explore and discover and share with each other what they do find.

Launch

Project the image of the Cantor Ternary Set and ask students what they see and how they see the pattern growing. You can record their different ways of seeing in the style of a number talk. You can see Jo introducing a visual pattern like she would a dot number talk in this video of the Border Problem: https://www.youcubed.org/resources/border-problem-video/ This is meant to help students become familiar with the pattern they're about to explore before thinking about it numerically.

As students get to know the pattern you might want to share some information on the pattern itself and the person behind it. It is called the Cantor Ternary Set after Georg Cantor, one of the first mathematicians to describe it. Cantor started the branch of mathematics called Set Theory, which investigates "sets" or collections of objects. A lot of his work dealt with infinite sets and he determined that there are different "sizes" of infinities. If this sounds weird to your students, they are not alone. It sounded weird to mathematicians in Cantor's day too, his work was extremely controversial at the time,



but it is now considered a key contribution to mathematics. The Cantor Ternary Set is an example Cantor used to think about properties of infinite sets and the students may end up exploring some of these properties as they come up with questions about Cantor's Ternary Set and how it grows.

After the introduction of the pattern, have students brainstorm questions they have about the pattern and its growth. Make a list of all the questions students come up with, posting them in a location where you can leave them up for the duration of this activity. This could be an area of a white board or some poster paper. Encourage students to come up with as many questions as they can, as they will want to have a variety of questions to explore. They will also continue to add questions to this list as they work and come up with new ones.
Some examples of questions they might come up with are:

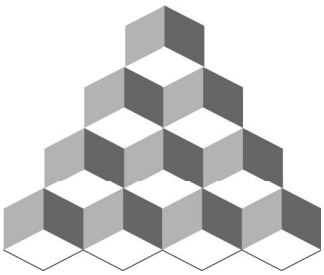
- How many black lines will there be in the 10th row?
- What is the total length of the black lines in the nth row?
- Will there ever be a row with no black lines?
- How many white spaces are there in any row?

Explore Part 1
Set students up to work in small groups. They have an investigation menu in all the questions they came up with as a class. Each group can choose to work on any of the questions they want for however long they feel is productive and interesting. They don't need to have answered a question fully to move on, but it is valuable for them to document their thinking even on partial solutions to the questions they work on. As they work, they may come up with other questions and they are encouraged to add those to the bank of questions for the whole class. Encourage students to work visually and ask questions that require visualizing, as visuals are a key part of mathematical thinking, one that is often forgotten.

Explore Part 2
In this next exploration time you can decide if you want to offer students some more related patterns to think about, or just let them continue working on what they started in the previous exploration session. Even if you do introduce the new patterns, it is good to give students a choice to continue working on the original pattern.

The second projection page below has images of two mathematical objects related to the Cantor Ternary Set visual, the Sierpinsky Carpet and the Menger Sponge. Show those to the students and share with them that they are often called generalizations of the Cantor Ternary Set. You can let students explore them and the question "What does it mean that these are called generalizations of the Cantor Ternary Set? How do you see that they are connected?" Invite students to see if the conjectures they came up with while working on the Ternary Set also apply to these other visuals. Encourage students to prepare a poster/visual showing how they see the Sierpinsky Carpet and Menger Sponge representing generalizations of the Cantor Ternary Set.

Regardless of whether you choose to introduce the additional visuals, ask students to represent their findings for each question in several different forms, such as expressions, graphs, pictures, etc. and color code them. They should put these on a poster to share with the rest of the class later.



Discuss

After the groups have had plenty of time to explore a few different questions, invite students to present their questions and answers to the rest of the class. We have purposefully given 50+ minutes to this phase because we want the class to become a true community of mathematicians where they not only have time to present finalized answers from their work in the explore phase, but also to present conjectures and partly-formed solutions. During this time they can build upon each other's work. As the groups present, you can keep a record of the different conjectures and unfinalized work shared. Different groups might add to others' work when they present, as they may have explored similar ideas. It is important to keep in mind, however, that not all questions raised need to be answered. It is okay to leave some open questions for students to keep with them and perhaps explore in the future.

If students are challenged with finding or sharing methods it is ok to introduce a method and share it with them. We recommend you do this by saying the method you are sharing is one you got from another student. This is an important message for them and a time when they should not see you as the expert.

Reflect

Ask students to reflect on their experience in their journal with a prompt like, "How is the growth of this pattern different from the ones you have explored so far?"

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Cantor Ternary Set

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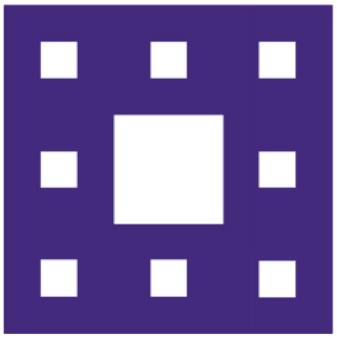
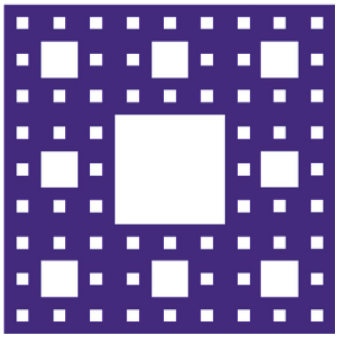
Sierpinski Carpet

Case 1

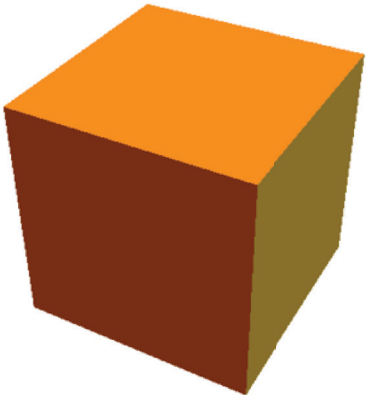


Case 2

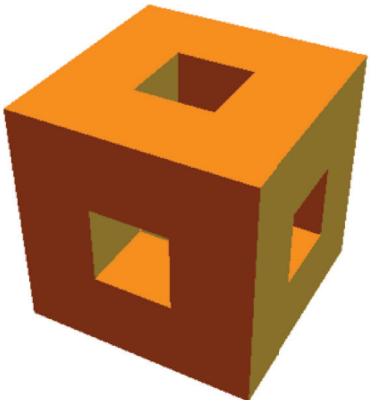


Case 3

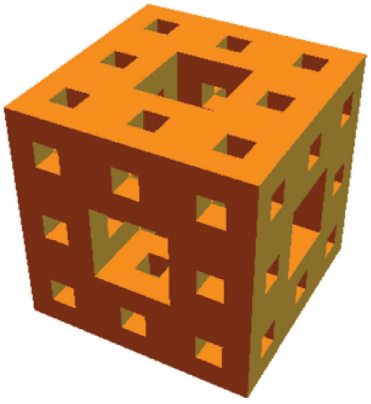
Menger Sponge



Case 1



Case 2



Case 3