

Painted Cube Week 4 - Day 1 & 2

Introduction

This is one of our most favorite go-to activities for making connections. It has been written about in several different journals, books and websites for many years and we are excited to share our Algebra 1 version here. The task is visual and tactile providing students different ways of constructing and seeing the pattern. The connections to surface area and volume, and to linear, quadratic, and cubic functions makes this activity one that is not to be rushed. That's why we say you need 2 – 3 days for students to fully explore the many different ways to build connections and understanding.

Connection to CCSS

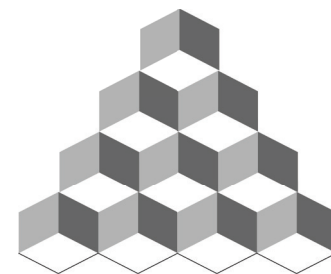
HSF.LE.A
HSF.LE.B, B.5
HSF.BF.A
HSG.GMD.B
HSA.CED.A, A.2

If you would like to learn more about the power of visual math for learning go to <https://www.youcubed.org/resource/visual-mathematics/>. We have a collection of videos for visual learning and making connections that you can share with students.

- <https://www.youcubed.org/resources/brain-crossing-video/>
- <https://www.youcubed.org/resources/tour-mathematical-connections/>
- <https://www.youcubed.org/resources/brains-think-math-visually-video/>

Agenda

| Activity | Time | Description/Prompt | Materials |
|----------|---------|---|--|
| Launch | 10 min | <ul style="list-style-type: none"> • Project the What do you see? Handout • Start a discussion asking, "What do you see?" Follow up with "How many cubes do you see?" | <ul style="list-style-type: none"> • What do you see? Handout (one copy per pair) |
| Explore | 30+ min | <p>Explore and gather data.</p> <ul style="list-style-type: none"> • Give students the Painted Cube handout and ask them to explore the questions for a 4x4x4 cube. | <ul style="list-style-type: none"> • Painted Cube handout • Sugar cubes, 126 count box for every 4 students • Poster paper (or table cloths) • Markers • Maths journal • Pencil/pen • Colored pencils |
| Discuss | 25+ min | <p>Create a class chart of findings.</p> <ul style="list-style-type: none"> • Invite students to record their findings in the class chart • Once all cells have been filled, work towards agreement on each entry. • Invite students to share patterns and conjectures they notice justifying them visually and numerically. | <ul style="list-style-type: none"> • Whiteboard • Colored markers/pens |

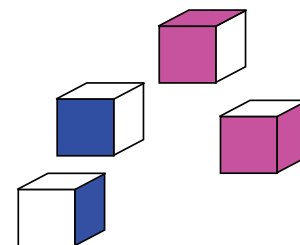


Agenda continued

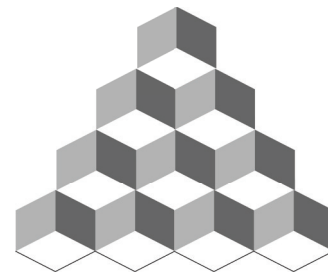
| Activity | Time | Description/Prompt | Materials |
|----------|---------|--|--|
| Explore | 20+ min | Explore growth graphically. <ul style="list-style-type: none"> • Display graph with all four functions (total volume, number of cubes with 0, 1, 2, 3 faces painted.) • Analyze the growth of the total number of cubes and number of cubes with 0, 1, 2, and 3 faces painted. | <ul style="list-style-type: none"> • Painted Cube Graph Handout • Sugar cubes • Multilink cubes |
| Discuss | 15+ min | Share conjectures about which graph represents the different patterns (total volume, number of cubes with 0, 1, 2, 3 faces painted.) | <ul style="list-style-type: none"> • Whiteboard |
| Extend | | <ul style="list-style-type: none"> • Design your own investigation using other shapes. • Describe other examples of visuals and situations that could be represented by the different graphs | <ul style="list-style-type: none"> • Maths journal |
| Reflect | 5 min | Ask Students: What did you learn about visuals, patterns, expressions, and graphs? | <ul style="list-style-type: none"> • Maths journal • Pencil/pen |

To the Teacher

This activity invites students to use multiple representations to explore the patterns of the number of faces painted on the unit cubes for different sized cubes. Students start the activity by building and gathering data so there is a need for them to keep good record of their findings. While groups explore, suggest they use a table as a way to organize the numbers they find. There is no need to give direction around what labels to use on the columns of the table because the layout of the table is not important during the exploring phase. Additionally, part of the learning involved in this activity is for students to decide how they will organize their data. To prepare for the discussion on the data students gathered, create a table on the whiteboard that will support them in their pattern finding.



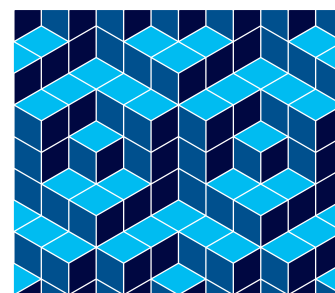
The exploration starts with building cubes of different sizes to gather data. This data is then organized, into a table and then graphs. Leave space in the discussion for students to name all the conjectures they have. When in discussion about the findings in the chart, ask students to show, explain, and justify their



conjectures visually. Prepare for this by arranging the board or poster paper in a way for students to draw, as they need to when discussing reasons. Once the discussion starts to include the table, expressions, and graphs, the visuals may not be present in the conversation. When this happens ask a question that encourages students to return to the visual. This is important because making sense visually and numerically and connecting this to more traditional representations like tables, equations, and graphs has the power to form or strengthen brain pathways.

Launch

Have students sit in groups of four. Start with an opportunity for students to look at a cube pattern to start visualizing and counting cubes in a different context. Say to students that you have an image you want to share with them. Pass out the What do you see? Handout, one copy per pair. Project the image and ask, "What do you see?" This is an opportunity for students to think creatively so take all of the responses that students offer. Then ask, "How many cubes do you see?" Give students some individual time to look at the cube pattern, and then tell them to share their thinking with their team. Bring the class back



together by asking what people got as the number of cubes in the pattern. If and when there are different numbers of cubes shared have students explain what they counted. When students are sharing remind them there will be differing ideas because of the openness of the question. You can also add that if we wanted a problem with just one answer we would need to ask a more specific question or include some parameters.

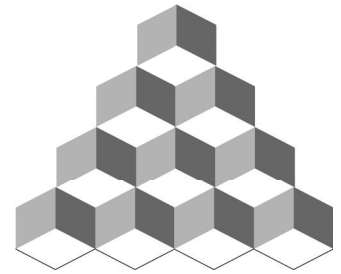
Next, transition to the painted cube activity. Explain to students that the activity involves cubes and that the conversation they just had about the different ways of looking at the cube pattern will help them think creatively. Ask students to clear their tables of everything. It is helpful to have lots of space to work. Have each group put a piece of poster paper on their tables. This will keep the sugar contained and give them a space to write ideas.

Launch the activity by sharing the scenario of dipping a 4x4x4 cube into paint and asking how many unit cubes would have 0 faces painted, 1 face painted, 2 faces painted, 3 faces painted, and so on. Share the materials that are available for them to use during their exploration. Encourage students to build with and color-code the sugar cubes. Keep this introduction to the problem brief by not seeking any answers to the questions posed. Support teams to ask and answer each other's clarifying questions.



Explore

There are so many patterns to think about when making sense of the painted cube problem. It is a good opportunity to establish reason for recording what students see in a chart. When teams are exploring with the sugar cubes give them space to come at the problem from any direction that makes sense to them. While groups work to build and gather data encourage them to record their findings as they go.



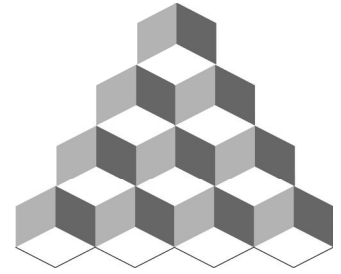
Start with posing the question, “If we took a 4x4x4 cube and dipped it into some paint, how many unit cubes would have no faces painted? One face painted, two faces painted, and three, four, five and six faces painted?”

Take some clarifying questions about what they will explore, then give each team of four a box of sugar cubes to use when modeling what would happen to a 4x4x4 cube being dipped in paint. Give teams plenty of time to explore through coloring and counting the cubes, and recording their findings in the chart in their journal. It is time to transition to a class discussion once every team has finished investigating a 4x4x4 cube and has started thinking about at least two other sized cubes (2x2x2, 3x3x3, 5x5x5, etc.)

Discuss

Write the table for the discussion on the whiteboard, if you write this table on the board before students have finished exploring, cover it so it won't influence students' organization. When you bring the class back together, invite students to share their findings and record them in the table. Encourage them to add anything they found in the table and if something is already in a cell or if there is not a place for it they should feel free to add columns or rows and use different colors to show disagreement. Discuss each entry until there is agreement. The bottom row does not have a title. The goal is to have someone notice the connection to the volume of a cube. If questions arise have students connect back to the visual to support their reasoning. Also connect back to the visual as you discuss the expression students create for any sized cube ($n \times n \times n$). How do the numbers in the expression relate to the cube? Be open to the different directions of the discussion. This will give you information about student understanding. If there are mistakes in the chart, allow students to discover and realize these mistakes and justify their reasoning. This will lead to interesting dialogue and creates a culture in the class of learning from mistakes and learning as a community from each other.

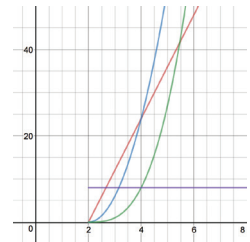
| | Size of cube | | | | | |
|------------------------------|--------------|-------|-------|-------|-------|-----------------------|
| # of painted faces on a cube | 1x1x1 | 2x2x2 | 3x3x3 | 4x4x4 | 5x5x5 | $n \times n \times n$ |
| 0 | | | | | | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| | | | | | | |



Explore

Once students have had an opportunity to complete and discuss the table. Tell students they will explore another representation of the patterns they have identified for the number of cubes with 0, 1, 2, and 3 faces painted and the total number of cubes. Display the graph and discuss the idea of making conjectures and what it might mean to check conjectures about graphs.

Give each pair of students one copy of the graph so that students have a copy they can analyze more closely. Ask students which graph represents which pattern or expression, using justifications based on the other representations. Encourage them to use the 3x3x3, 4x4x4, or 5x5x5 cubes they built when thinking about a visual explanation, to support this you might give students more sugar cubes or some multilink cubes. Consider drawing a visual on the board of a 4x4x4 or 5x5x5 cube.



Discuss

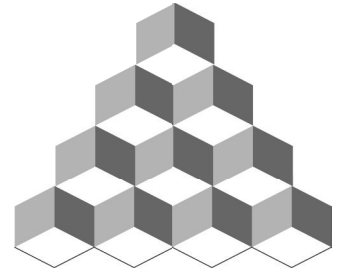
Invite teams to share their strategies for connecting the different representations. Highlight the connections students make between the representations. Remind students that connecting across multiple representations, including the ways they see math visually, is important for learning. For more information on the power of visual math go to <https://www.youcubed.org/resource/visual-mathematics/>. If you would like to share a video about brain crossing go to <https://www.youcubed.org/resources/brain-crossing-video/>.

Share with students the names of the different functions (linear, quadratic, cubic), ask them to share ideas about how each function got its name, and to make connections between the names of the function and its visual representation. Bring more connections into the conversation by asking a question like, "What will happen to these graphs as the size of the cube increases? How does that relate to the visual? Which pattern is growing more quickly than the others? Why? What do these intersection points mean? What would happen if we zoomed in on the graph or zoomed out?" Tell students they will continue to learn about these different kinds of growth and their equations and graphs throughout the course.

Give students the second graph handout. Ask students: What do they notice about the difference between the two graphs on the handouts? Why on one graph handout do the x values start at 2? What do you notice about the shape of each graph and what happens to the y values when the x values are less than 2? Highlight the shape of the graphs for different types of growth: linear, quadratic, and cubic. Bring students back to a conversation about the domain and range of a function and what set of numbers is appropriate for these values.

Extend

- Design your own investigation using other shapes (rectangular prism, square based pyramid.)
- Brainstorm other examples of visuals and situations that could be represented by the different graphs.



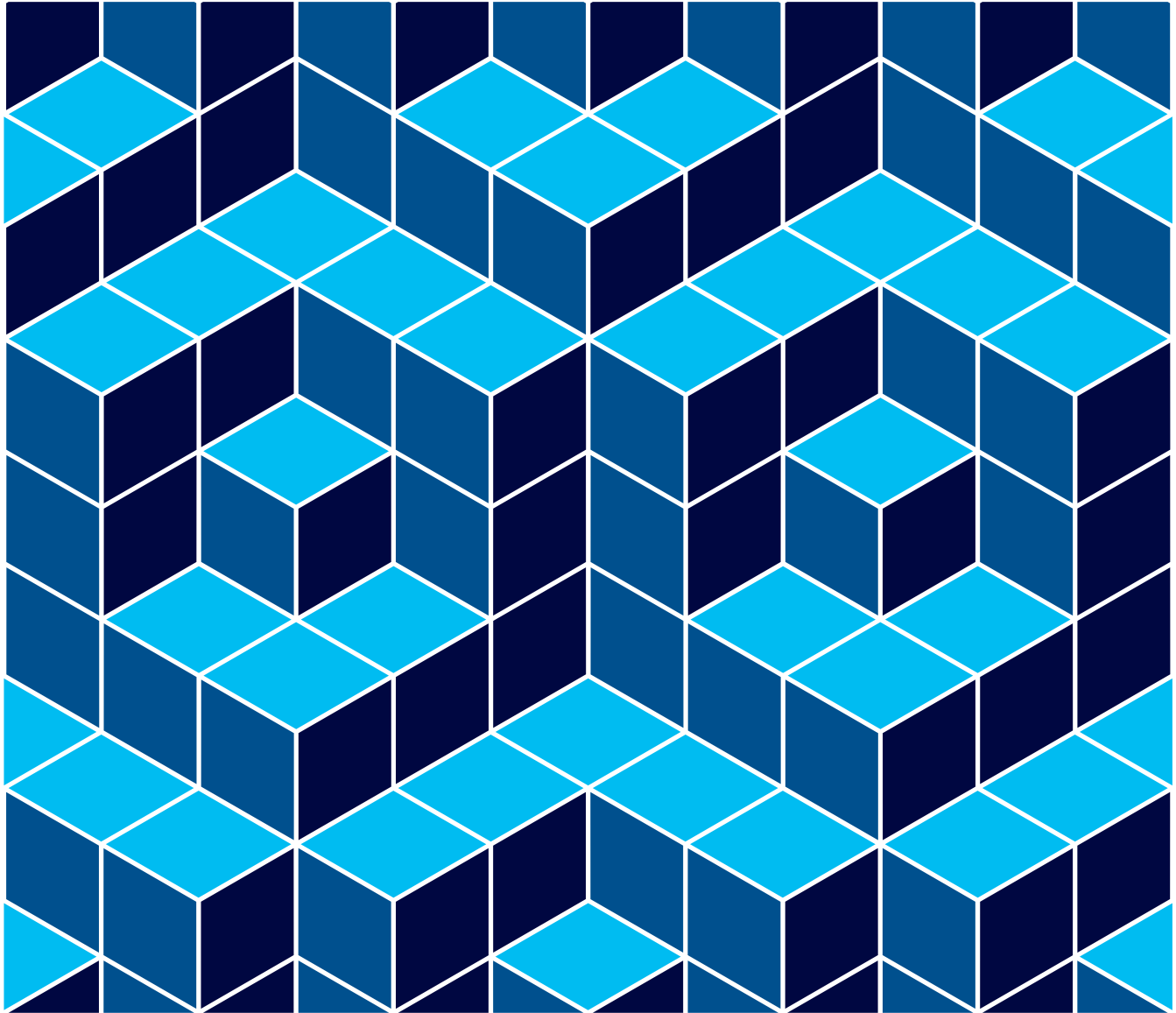
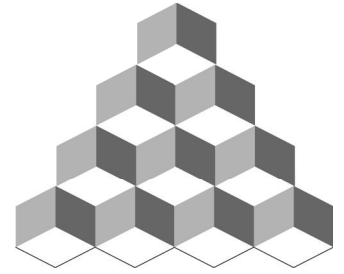
Look-Fors

- How are students counting cubes? When students use different counting strategies they can visualize patterns and make more connections to expressions and other representations of the pattern. Encourage students to share their different ways of seeing which will make space for students to see different ways of counting.
- How are students recording their findings? With a goal of generalizing patterns and writing equations notice how students are recording their findings. While students are working you might notice students are recording their findings and yet not organizing them so that they can see patterns emerge. If this seems to be happening in every group it's okay to wait until you bring the class together. At this point students will share their findings on a class chart during the discussion.
- How are students justifying their conjectures? When students are discussing the findings in the table listen to how they are explaining their conjectures and how students are testing and justifying them. Students will likely use the representations they are most comfortable with which is important to make space for. It is also a good idea to ask about how the conjecture could be justified visually so that students are encouraged to connect to the visual.
- How are students using the visuals to justify their thinking? When students are giving reasons for their ideas notice how they use the visuals in their explanations. If you notice the visuals are only discussed when students are sharing their findings, encourage them to make connections between the visual and the expression and/or the visual and the graph. Seeing the visual connections will give students a concrete image of the difference between the growth, expressions/equations, and graphs that they will be able to recall throughout the year.
- How are students connecting the graphical representations of the growth of the number of unit cubes painted with 0, 1, 2, and 3 faces painted? When thinking about the graphical representations of growth patterns take notice of how they are describing the connections. Encourage students to show their thinking visually with color-coding, arrows, circling, etc.
- How are students generalizing their patterns? When students talk about generalizing take notice of how they are finding expressions/equations for the number of cubes with 0, 1, 2, 3 faces painted. If students are only using what they notice about the numbers in the table ask them to think about it visually and discuss where the growth is happening so they are connecting the expressions to the different parts of the larger cube that are changing.

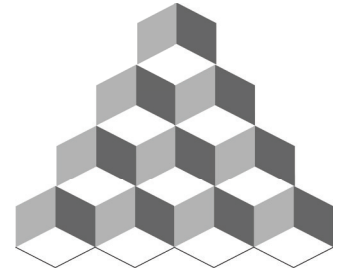
Reflect

What did you learn about visuals, patterns, expressions, and graphs?

What do you see?
Handout



Painted Cube Handout



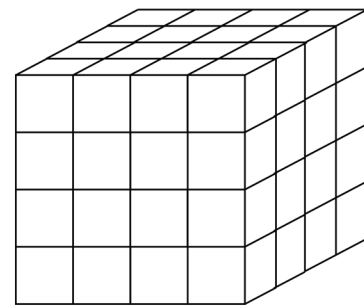
Case 1



Case 2



Case 3



Case 4

Task Instructions

Imagine that we paint a $4 \times 4 \times 4$ cube blue on every side.

How many of the small cubes have no paint on them?

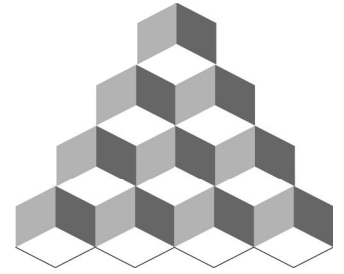
How many have 1 blue face?

How many have 2 blue faces?

How many have 3 blue faces?

How many unit cubes have no faces painted, 1, 2, or 3 faces painted in a cube of any size?
Think visually.

Painted Cube Graph Handout



Painted Cube Graph Handout

