



# Day 5: Growing Shapes

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## Introduction

This lesson includes a video and a wonderful task that you may want to extend into day 6!

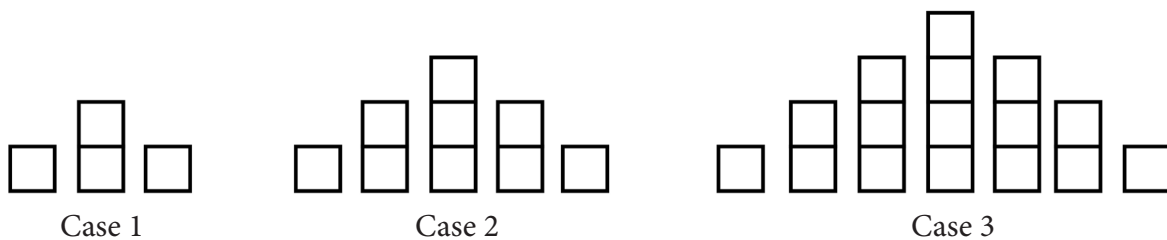
## Video

This may be your students' favourite video! It gives new research evidence on the importance of mistakes for brain growth and the need for challenge and thinking differently. It includes some really nice animations with the Stanford students.

## Activity: Growing Shapes

This is one of my favourite tasks as it asks students to look at growing cases and describe how they see the shapes growing. Usually students are given growing cases with the question: How many are in the 100th case? or the nth case? We have found it really helpful to ask a different question first – “How do you see the shapes growing?” “Please don't think about numbers or algebra just the growth of the shape.” This usually prompts eight or nine different ways of seeing the growth of the shapes. Ask students to think of this question alone before moving to groupwork, if you ask them to start in groups most people will end up seeing it in the 'same' way.

### How do you see the shapes growing?



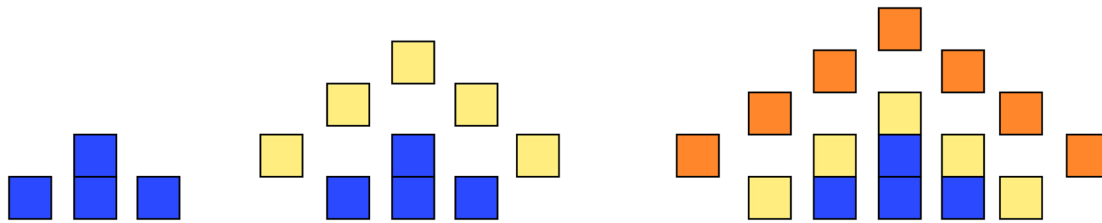
After asking the students: How do you see the shapes growing? Give students some time to think alone. You can find out when people are ready by using a number talk strategy: Ask students to show you a “quiet thumb” when they have had enough time. Putting hands up puts other students under pressure, so a quiet thumb – one held by their chests, not waived in the air, works better.

I have asked many people – teachers, students, and others how they see the shapes grow. This has shown me that there are many different ways of seeing the growth, and people are fascinated to see the different ways.

After you have asked students how they see the shape growing invite different students to the front board to share their ways of seeing with the class. A nice way to do this is to project the shape onto the front board, so that students can draw around it. I always name the different methods, with the students' name and a name for the method. For example some people see the shape growing as additional squares on the top of each column:



# Day 5: Growing Shapes

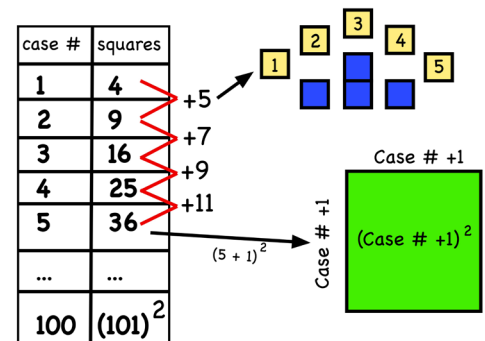


One of my students named this the raindrop method.

When I trialled this with Year 7 they saw the shapes grow in 8 different ways. The students were a little un- sure about names at first but I helped them with the first ones and they got the hang of it. After we looked at the different ways people saw the shapes I gave students our handout and asked them to talk in their groups, finding out all the ways people saw the shape growing and then showing each method on the handout. You may like stu- dents to show one or more methods on a classroom poster. I then asked the groups of students to use one of their methods to show what the 100th case looked like.

After they had thought visually about the 100th case I asked them to think about numbers, if they hadn't already – how many would the 100th case have? What would it look like? And how do the numbers relate to the visual solution?

Ask the students to make a table with all of their results. Ask students to use both the number pattern they see in their table and their visuals to predict how many squares there would be in the 4th and 5th case. Even though they have already thought about the hundredth case adding more rows to their table can help them see the functional growth in both the numbers and the shape. Keep encouraging students to connect the numbers and visuals. Ask them where they see the extra squares in their pictures and in the numbers in the table they have made.



This task has a low floor and a high ceiling – the low floor means anyone can see how the shape is growing, but it extends to high levels and the function that is represented by the shapes is a quadratic function. Tasks such as this are a really nice way to introduce students to variables. If there is time and if it is appropriate for your stu- dents and you would like to introduce them to variables, ask students, in groups to express the functional growth first in words, eg

Could you tell me how many squares there would be in any case, if I just told you the case number? Eg if I told you it was case 500 how many squares would there be? Write a sentence to describe how many squares there would be with any case number.

This can lead to asking students to use a variable instead of the words.



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This is a rich activity that prompts wonderful discussions between students, allowing them to see an algebraic representation and develop meaning for it. Forty minutes is probably too short a time for it, so if your lessons are short you may want to take it into 2 lessons. Usually when growing cases are given out, they have the instruction: How many are in the 100th case or the  $n$ th case? with no discussion of the different visual ways people see the cases. But when students can share and discuss the different ways they see shapes, they develop meaning and understanding of the functional growth, that doesn't come when students only focus on numbers.

Activity	Time	Description/Prompt	Materials
Day 5 Video: Mistakes	3 min	Video <a href="https://www.youcubed.org/wim-day-5/">https://www.youcubed.org/wim-day-5/</a>	
Growing Shapes	30 min	<ol style="list-style-type: none"> <li>1. How do you see the shapes growing? ask students to think alone at first</li> <li>2. Ask students to share their methods, drawing them on the board.</li> <li>3. Name the methods, with the students' name and possibly a method name e.g. the raindrop method</li> <li>4. Ask students to find out how their group saw the shapes growing</li> <li>5. Ask students to think about the 100th case</li> <li>6. Ask the students to construct tables and link their numbers and visuals.</li> <li>7. Possibly ask them to think about generalization and variables</li> </ol>	<ul style="list-style-type: none"> <li>• Paper, pencil/pen</li> <li>• Coloured pencils/markers</li> <li>• Shapes Task, page 4. One copy per student</li> <li>• Shapes Task for display, page 5. One copy</li> <li>• One page of chart paper per group if you want to have classroom posters</li> </ul>
Group Presentations	10 min	Ask students to share any patterns or other interesting observations	
Closing	5 min	Review the key concepts: Mistakes Grow Your Brain!	

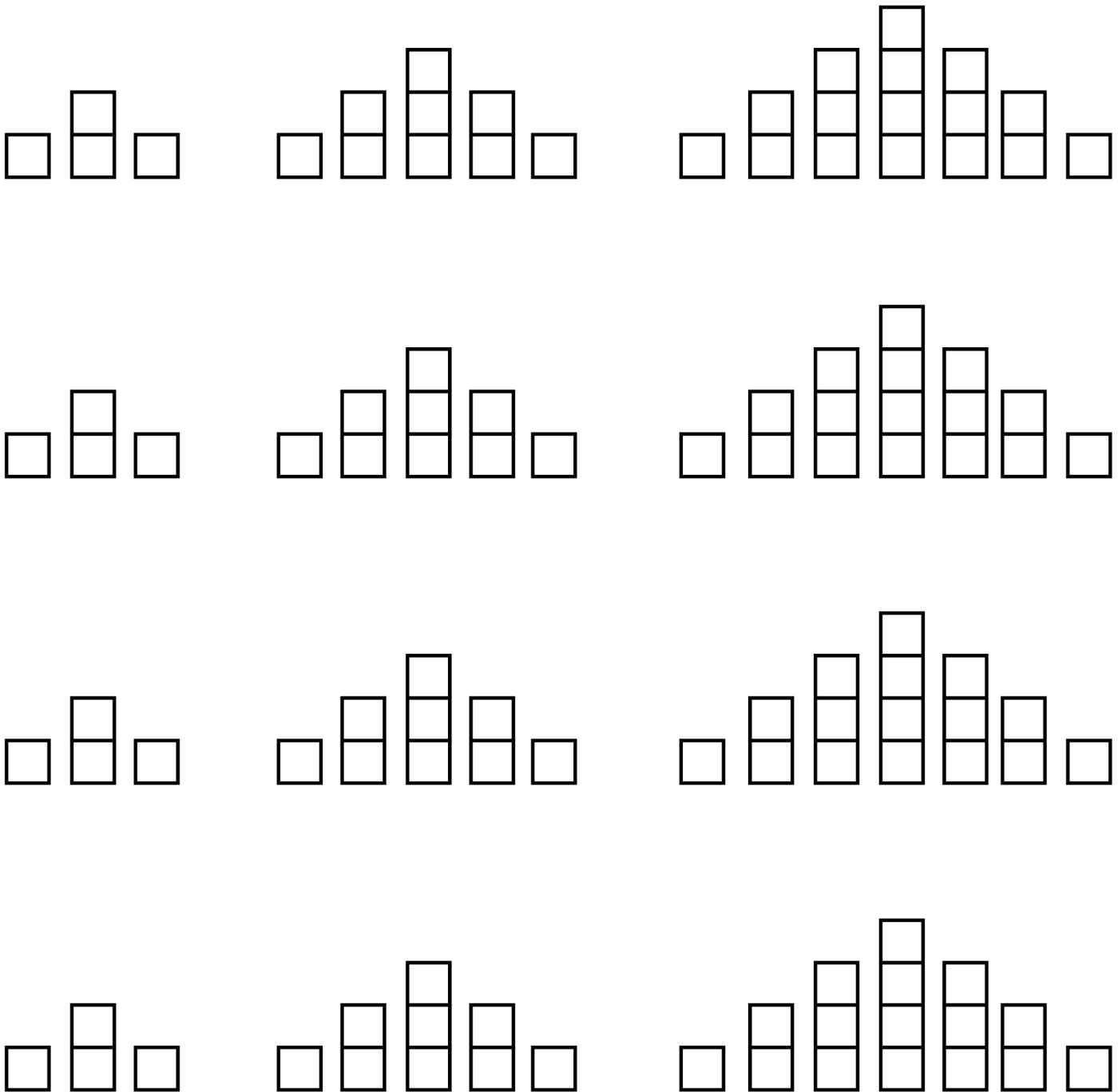
### Extensions:

- Show an algebraic expression and a visual proof for the number of squares in the  $n$ th case
- Which case would have 289 squares?



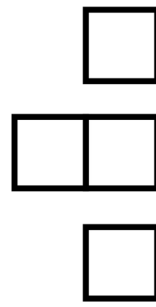
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Different colours can be helpful in showing how people see the shapes!





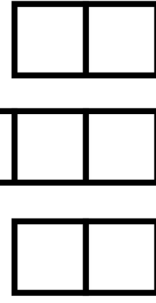
How do you see the shapes growing?



Case 1



Case 2



Case 3

