Extract from Chapter 1. What is Math? from What's Math Got To Do With It: How Teachers and Parents Can Transform Mathematics Learning and Inspire Success (Penguin). pages 25, 27 & 29

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Problem solving is at the core of mathematicians' work, as well as the work of engineers and others, and it starts with the making of a guess. Imre Lakatos, mathematician and philosopher, describes mathematical work as "a process of 'conscious guessing' about relationships among quantities and shapes."13 Those who have sat in traditional math classrooms are probably surprised to read that mathematicians highlight the role of guessing, as I doubt whether they have ever experienced any encouragement to guess in their math classes. When an official report in the UK was commissioned to examine the mathematics needed in the workplace, the investigator found that estimation was the most useful mathematical activity. Yet when children who have experienced traditional math classes are asked to estimate, they are often completely flummoxed and try to work out exact answers, then round them off to look like an estimate. This is because they have not developed a good feel for numbers, which would allow them to estimate instead of calculate, and also because they have learned, wrongly, that mathematics is all about precision, not about making estimates or guesses. Yet both are at the heart of mathematical problem solving.

After making a guess, mathematicians engage in a zigzagging process of conjecturing, refining with counterexamples, and then proving. Such work is exploratory and creative, and many writers draw parallels between mathematical work and art or music. Robin Wilson, a British mathematician, proposes that mathematics and music "are both creative acts. When you are sitting with a bit of paper creating mathematics, it is very like sitting with a sheet of music paper creating music."15 Devlin agrees, saying that "Mathematics is not about numbers, but about life. It is about the world in which we live. It is about ideas. And far from being dull and sterile, as it is so often portrayed, it is full of creativity."

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Something else that we learn from various accounts of mathematicians' work is that an important part of real, living mathematics is the posing of problems. Viewers of A Beautiful Mind may remember John Nash (played by Russell Crowe) undergoing an emotional search to form a question that would be sufficiently interesting to be the focus of his work. People commonly think of mathematicians as solving problems, but as Peter Hilton, an algebraic topologist, has said: "Computation involves going from a question to an answer. Mathematics involves going from an answer to a question."19 Such work requires creativity, original thinking, and ingenuity. All the mathematical methods and relationships that are now known and taught to schoolchildren started as questions, yet students do not see the questions. Instead, they are taught content that often appears as a long list of answers to questions that nobody has ever asked. Reuben Hersh puts it well:

> The mystery of how mathematics grows is in part caused by looking at mathematics as answers without questions. That mistake is made only by people who have had no contact with mathematical life. It's the questions that drive mathematics. Solving problems and making up new ones is the essence of mathematical life. If mathematics is conceived apart from mathematical life, of course it seems—dead.20

Bringing mathematics back to life for schoolchildren involves giving them a sense of living mathematics. When students are given opportunities to ask their own questions and to extend problems into new directions, they know mathematics is still alive, not something that has already been decided and just needs to be memorized.

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Mathematics is a performance, a living act, a way of interpreting the world. Imagine music lessons in which students worked through hundreds of hours of sheet music, adjusting the notes on the page, receiving checks and crosses from the teachers, but never playing the music. Students would not continue with the subject because they would never experience what music is. Yet this is the situation that continues, seemingly unabated, in mathematics classes.

Those who use mathematics engage in mathematical performances. They use language in all its forms, in the subtle and precise ways that have been described, in order to do something with mathematics. Students should not just be memorizing past methods; they need to engage, do, act, perform, and problem solve, for if they don't use mathematics as they learn it, they will find it very difficult to do so in other situations, including examinations

Maryam Mirzakhani is a mathematician at Stanford who was recently featured in newspapers across the world by winning the Fields Medal, the most prestigious prize in mathematics. When news of the award spread, The Telegraph, Britain's best-selling newspaper, asked me to write an article on Maryam's work. I was happy to do so,22 and I told the newspaper that just weeks before I had chaired a PhD defense for one of Maryam's students in Stanford's math department. A PhD defense is the occasion when a student defends her PhD dissertation to a committee of scholars. That day I sat with the other members of the committee, all mathematicians, and watched as Maryam's student, a young woman, paced the room, showing visual representations and sharing conjectures, using mathematics creatively to connect different ideas. Many times in the defense she was asked a question to which she replied, "I don't know." This answer was perfectly reasonable and accepted by the mathematicians on the committee because she was exploring new territory to which no one had answers. This struck me as highly significant because school children everywhere would be shocked to see that high-level mathematics involves such creativity and uncertainty.

The erroneous thinking behind many school approaches is that students should spend years being drilled in a set of methods that they can use later. The mathematicians who oppose change are most concerned about the students who will enter graduate programs in mathematics. At that point students will encounter real mathematics and use the tools they have learned in school to work in new, interesting, and authentic ways. But by this time most students have given up on math. We cannot keep pursuing an educational model that leaves the best and the only real taste of the subject to the end, for the rare few who make it through the grueling years that precede it. If students were able to work for at least some of the time in the ways mathematicians do—posing problems, making conjectures using intuition, exploring with and refining ideas, and discussing ideas with others—then they would not only be given a sense of true mathematical work, which is an important goal in its own right, they would also be given the opportunities to enjoy mathematics and learn it in the most productive way.